DEUTERON STRIPPING REACTIONS ON CARBON AND OXYGEN: A COMPARISON OF THEORY AND EXPERIMENT

By

HAROLD ARTHUR MOORE

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I. INTRODUCTION

The term "stripping" seems to have been coined by Serber (1) when he attempted to explain the work of Helmholtz, et al (2), who had found that the bombardment of various targets by 190 mev deuterons resulted in a beam of neutrons of half-width of about ten degrees in the forward direction. This beam was of a fairly well-defined energy in the region of 90 Mev. Serber explained this beam by means of a simple model in which the deuteron incident on the target nucleus has the proton stripped off and captured, the neutron carrying away half the energy of the incident deuteron. Serber's theory applies equally well to the case in which the neutron is captured and the proton carried away.

Others found similar results that agreed with Serber's theory. Hadley, et al (3), and Bruechner, et al (4), both measured the spectrum of neutrons from a Be target using 190 Mev deuterons and found agreement with Serber's theory to within experimental accuracy. Chupp, et al (5), made careful experiments on the spectrum of protons from the (d,p) reaction with the same deuteron beam and a Cu target. They also obtained good agreement.

Other experiments were done between 1949 and 1951 (e.g., Allen, et al (6), Ammiraju (7), Gove (8), Schecter (9)) with energies between the values of 14 and 20 Mev, measuring the angular distributions of emitted particles of a greater or lesser unresolved energy range, and the results compared with Serber's theory. The half-widths and even the shapes of the curves showed violent fluctuations with the part of the energy spectrum selected. Gove, for example, using 14 Mev deuterons on a number of targets (C, Al, etc.) found definite peaks for the carbon reaction which did not fit Serber stripping.

The greatest difficulty that lay in analyzing the data obtained from deuteron stripping prior to 1951 was the poor energy resolution of the emergent nucleon beams; until the energy resolution was improved to the point that definite nucleon groups could be identified it was not definitely known that each group represented an energy level in the final nucleus. Burrows, et al (10), were the first to obtain resolved angular distributions representing the protons from the ground state reaction in O^{16} using 8 Mev deuterons. These distributions showed concentrations of protons at small angles to the deuteron beam expected on Serber's theory, but the curve for the ground state reaction had an unexpected decrease very close to the forward direction. The curve for the first excited state, however, continued to rise with decreasing angle down to the smallest angles measured. This can be seem in the O^{16} stripping curves shown in Section IV. Many similar results were obtained after this by others with greater accuracy, especially by the Liverpool group with which Burrows was associated (Rotblat, Gibson, et al). It was data from this group that Butler (11) first analyzed using his stripping theory, satisfactorily explaining the forward maximum in the angular distributions.

The stripping theory that Butler developed has been an extremely useful tool in nuclear spectroscopy. The most important application of his stripping theory lies in the fact that it enables the assigning of spins and parities to the levels of the final nucleus of the stripping reaction. It is not always possible, however, to assign a single spin to a level since the selection rules may be satisfied by more than one value.

Since 1951, the Butler theory has been quite useful in analyzing data. Holt and Marsham (12), in particular, did extensive stripping experiments with many elements. They were able to assign spins and parities to a number of elements ranging from Be^9 to Sr^{88}, managing to resolve proton groups which represented as high as the eighth and ninth excited states of isotopes. Their results for nuclear spins and parities fit well with the shell model theory of the nucleus.

Others who obtained good results were Schecter and King and Parkinson (13). King and Parkinson analyzed the Cl^{35} (d,p) reaction, subtracting off the background, and obtained an excellent fit with theory. Schecter, one of the first to use the Butler analysis obtained good results with a relatively high deuteron energy of 20 Mev in the (d,n) reaction of Be^9 . Evans, et al (14), also obtained good results.

Although the Butler theory has had extremely good success it has had failures. The theory breaks down for the case of the neutron angular momentum $l_n = 0$ when the energy of the neutron, E_n, is approximately equal to the binding energy of the deuteron. This means a negative Q value since $E_n = -(Q + e_d)$, where e_d is the binding energy of the deuteron. An example of this is the analysis of Middleton, et al (15), of the C^{12} (d,n) reaction. The results are completely anomalous. The nearest fit is obtained with an $l_p = 1$ curve (it should be remembered that the stripping theory holds for either a (d,n) or a (d,p) reaction). The otherwise known spins and parities of the levels in this case taken together with the selection rules require that $l_p = 0$. But not all the failures are for the negative Q values. Evans and Parkinson (16) in taking the angular distributions for the B^{10} (d,p) reaction noticed that the first excited state reaction for which Q is 7.13 Mev does not fit the required $l_n = 1$ curve. Also there is an energy dependence that is odd in this distribution. The curve moves in closer and closer to smaller angles as the energy is decreased.

Another puzzle is that of the F^{19} (d,p) reaction. The stripping reaction fixes the spin of the ground state of F^{20} as unity. This value, however, is in disagreement with data from beta decay.

The Butler theory is not the only theory that has been presented for stripping reactions. The others, notably the ones of Bhatia, et al (17), and Daitch and French (18) have been successful but in essence are the same as the Butler theory, the only difference being in the assumption of the region of interaction or extent of interaction of the captured particle with the target nucleus. The "Bhatia curve" gives essentially the same type of distribution as the "Butler curve" but the radius required to fit the Bhatia theory is always approximately the order of 1×10^{-13} cm larger than that required for the Butler theory.

The Daitch and French theory is the same as the Butler theory except that there is a multiplying factor which removes one of the zeroes of the Butler distribution.

The Butler theory completely neglects such effects as coulomb interaction and resonances. Tobocman and Kalos (19) have extended the theory to include interactions with the target nucleus. Others, such as Glauber (20), who included diffractive effects, and Thomas (21), who writes the general matrix element for the interaction to include other effects, have not altered the basic idea of stripping.

Even though the Butler theory has had remarkable success in explaining stripping phenomena and correlating with nuclear shell models (up to O^{16} the agreement is complete) the fact remains that the agreement with experimental data is good only in a very limited region of the forward peak of the angular distributions. Most of the angular distributions have been taken at a single deuteron energy and it has never been shown that there is any consistency between theory and experiment at different deuteron energies.

It is the purpose of this investigation to attempt to determine the energy dependence of the angular distribution in stripping reactions and to investigate the internal relationships of the simple (Butler type) stripping theories so as to develop a validity criterion for them. Further, it is our purpose to apply such a validity criterion to pertinent data in order to determine whether the energy dependence predicted by the theory is correct.

II. STRIPPING THEORIES

A. Simple Model.--Throughout this work we shall consider only the (d,p) reaction. Also, we shall let $\hbar = 1$.

A deuteron of momentum k_d is incident on a target nucleus, A. The deuteron, passing close enough to the nucleus, disintegrates. The neutron is captured and the proton proceeds with momentum k_p. The assumption is made that the proton does not interact with the nucleus. This non-interaction characterizes the reaction known as the stripping process. In a non-stripping or compound nucleus process the proton would be absorbed into the target along with the neutron and then re-emitted.

The neutron proceeds toward the nucleus with a momentum given by

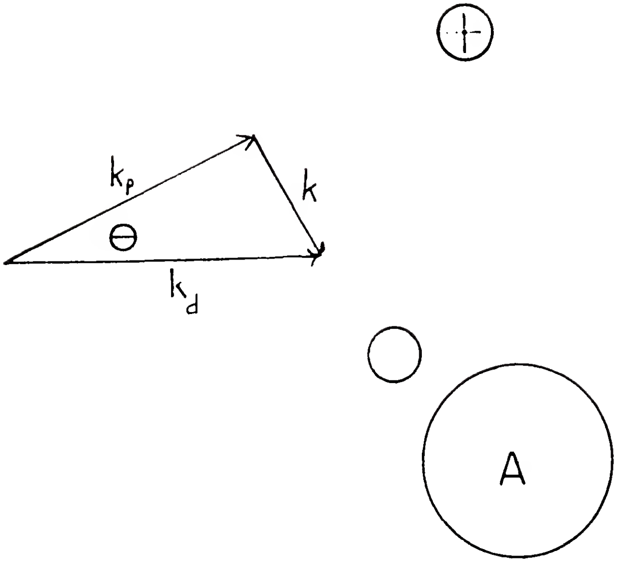
$$\vec{k} = \vec{k}_d - \vec{k}_p \tag{1}$$

or

$$k^2 = k_d^2 + k_p^2 - 2 k_p k_d \cos \theta , \tag{2}$$

where θ is the angle between the direction of the incident deuteron and the direction of the emergent proton (see Fig. 1).

According to the shell model theory of the nucleus, if the neutron is to be captured by the target nucleus it must be captured with quantized angular momentum l_n such that the conservation of angular momentum law is satisfied. That is, Fig. 1.--Simple stripping model.


k_p
k
Θ
k_d
A

$$\vec{J}_i + \vec{\ell}_n + \tfrac{1}{2} = J_f \,, \tag{3}$$

where J_i is the spin of the initial nucleus, J_f is the spin of the final nucleus and 1/2 is the spin of the neutron.

Classically, for those neutrons having impact parameter R equal approximately to the radius of A, we should have

$$kR = \ell_n \quad . \tag{4}$$

Thus, for a fixed k_d and k_p it would seem that equation (2) could be satisfied only for some definite angle θ_{l_n}. However, even classically some captured neutrons will have smaller impact parameters than R and quantum effects will permit further latitude. This means that the angular distribution will not be concentrated at a single angle but will actually be a distribution. The cross-section for the reaction will contain a factor representing the probability amplitude that the neutron possessing linear momentum k is found at the nuclear surface with angular momentum l_n..

B. General Expression for the Cross-section.--The formal theory for the stripping cross-section is derivable from the approximation methods for time-dependent problems in quantum mechanics.

Consider a deuteron incident on a target nucleus which is represented by a potential region V(r). The proton proceeds unscathed at an angle θ to the forward direction while the neutron is captured into the potential region in a bound state. The differential cross-section is proportional to the transition probability per unit time per unit flux between the initial and final state of the deuteron as it enters and leaves the potential region. Since the incident flux is proportional to the velocity of the deuteron and hence to the momentum k_d, we can write

$$\sigma(\theta) \propto \frac{\omega}{k_d} , \tag{5}$$

where ω is the transition probability per unit time between states.

The transition probability, ω, is proportional to the density of final states of the emergent proton, $\rho(k_p)$, and the square of the matrix element of the perturbation, H', between the final and initial states of the system. Then,

$$\omega \propto \rho(k_p) \left| H'_{fi} \right|^2 , \tag{6}$$

where

$$H'_{fi} = \int \psi_f^* V \psi_i \, d\tau . \tag{7}$$

The ψ_f and ψ_i are the wave functions of the final and initial states of the system. The cross-section can now be written in the form

$$\sigma(\theta) \propto \frac{\rho(k_p)}{k_d} \left| H'_{fi} \right|^2 . \tag{8}$$

Since $\rho(k_p)$ is proportional to the final momentum of the proton, k_p,

$$\sigma(\theta) \propto \frac{k_p}{k_d} \left| H'_{fi} \right|^2 . \tag{9}$$

Assuming the nuclear spin of the initial nucleus is zero, we can write, following Daitch and French,

$$\sigma(\theta) = \frac{m'_d m'_p}{3(2\pi^2)} \frac{k_p}{k_d} \frac{(J_f + \frac{1}{2})}{(2l_n + 1)} \sum_{m_p, m_d, m_n, m} \left| H'_{fi} \right|^2 . \tag{10}$$

Here, m_d, m_p, m_n are the spin magnetic numbers for the deuteron, proton and neutron; m is the orbital magnetic quantum number for the captured neutron. Since the incident particle and target nucleus are not polarized, σ is averaged over the initial states and summed over the final states. The sum over the spin quantum numbers gives simply a factor of three such that the cross-section becomes

$$\sigma(\theta) = \frac{m'_d m'_p}{2\pi^2} \frac{k_p}{k_d} \frac{(J_f + \frac{1}{2})}{(2l_n + 1)} \sum_{m_p, m_d, m_n, m} \left| H'_{fi} \right|^2 , \tag{11}$$

where m'_d and m'_p are the reduced masses of the deuteron and proton respectively.

It is necessary now to arrive at an expression for the matrix element. We assume the neutron is captured under the influence of a potential (assumed central) given by $V(r)$ into the one-particle state with space dependence

$$\psi_l^m(\vec{r}) = R_l(r)\, Y_{lm}(\theta, \phi) \tag{12}$$

and with binding energy

$$E_n = \frac{t^2}{2m_n'} \quad , \qquad (13)$$

where m_n' is the reduced mass of the neutron.

We assume that as an approximation the initial wave function of the system is given by

$$\Psi_i = \Psi_d(s) e^{i\vec{k}_d \cdot \vec{r}_d} \quad , \qquad (14)$$

where s is the internal coordinate of the deuteron given by

$$\vec{s} = \vec{r}_p - \vec{r} \qquad (15)$$

and

$$\vec{r}_d = \frac{1}{2}(\vec{r}_p + \vec{r}) \qquad (16)$$

is the center of mass coordinate of the deuteron. The final wave function of the system is given by

$$\Psi_f = e^{i\vec{k}_p \cdot \vec{r}_p} \varphi_\ell^m(\vec{r}) \quad . \qquad (17)$$

The matrix element is then

$$H_{fi}' = \int e^{-i\vec{k}_p \cdot \vec{r}_p} \varphi_\ell^{\overset{*}{m}}(\vec{r}) V(r) \Psi_d(\vec{s}) e^{i\vec{k}_d \cdot \vec{r}_d} d\vec{r} d\vec{p} \,. \qquad (18)$$

If we make the substitution

$$\vec{r}_p - \vec{r} = \vec{s} \qquad (19a)$$

and

$$\vec{r}_d = \vec{r} + \frac{\vec{s}}{2} \quad , \tag{19b}$$

we arrive at

$$H'_{fi} = \int e^{-i\left(\vec{k}_p - \frac{\vec{k}_d}{2}\right)\cdot\vec{s}}\, \psi_d(\vec{s})\, d\vec{s}$$

$$\times \int e^{i(\vec{k}_d - \vec{k}_p)\cdot\vec{r}}\, V(r)\, \varphi_\ell^{*m}(\vec{r})\, d\vec{r} \; . \tag{20}$$

To understand equation (20) more clearly we refer to the simple model of the stripping process (section A) and consider it as a two stage process. Firstly, the deuteron disintegrates and, secondly, the neutron is captured by the target nucleus. When the deuteron disintegrates the neutron has momentum K with respect to the center of mass of the deuteron. This momentum is given by (see Fig. 2),

$$\vec{K} = \vec{k}_p - \frac{\vec{k}_d}{2} \tag{21}$$

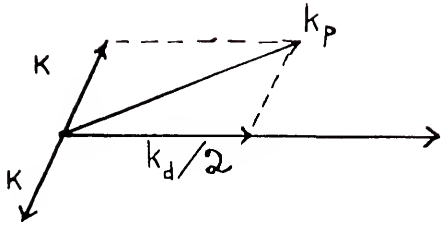
or

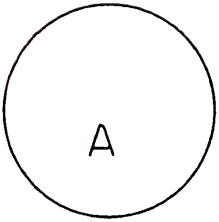
$$K^2 = k_p^2 + \frac{k_d^2}{4} - k_p k_d \cos\theta \quad . \tag{22}$$

The momentum of the captured neutron is

$$\vec{k} = \vec{k}_d - \vec{k}_p \quad . \tag{23}$$

Fig. 2.--Stripping model showing momentum K.


k_p
K
K
$k_d/2$


A

Thus we see that we can rewrite equation (20) in the form

$$H'_{fi} = D(K)\,G(k) \quad , \tag{24}$$

where

$$D(K) = \int \psi_d(\vec{s})\, e^{-i\vec{K}\cdot\vec{s}}\, d\vec{s} \tag{25}$$

and

$$G(k) = \int \varphi_l^{*m}(\vec{r})\, V(r)\, e^{i(\vec{k}_d - \vec{k}_p)\cdot\vec{r}}\, d\vec{r} \;. \tag{26}$$

D(K) expresses the probability that the momentum K will be obtained from the disintegration of the deuteron. G(k) expresses the probability that a neutron of momentum k arrives at the nuclear surface with the proper angular momentum l_n to be captured.

The cross-section is then given by

$$\sigma(\theta) \propto \frac{k_p}{k_d} \left| D(K)\,G(k) \right|^2 \quad . \tag{27}$$

Before equation (27) can actually be used for computation a correction must be made to the neutron momentum. The integral in equation (26) is taken over the neutron coordinates and not over the coordinates of the center of mass of the whole system. If the target nucleus were infinitely heavy this would be all right, but since we cannot assume this we must correct for it. Allowing for the finite size of the nucleus the momentum of the neutron is seen by the center of mass of the final nucleus is

$$\vec{k} = \vec{k}_d - \frac{m_i}{m_f}\vec{k}_p \qquad (28)$$

or

$$k^2 = k_d^2 + \left(\frac{m_i}{m_f}\right)^2 k_p^2 - \frac{2m_i}{m_f} k_p k_d \cos\theta . \qquad (29)$$

(see appendix for further treatment).

C. The Deuteron Factor, D(K).--As was mentioned in the previous section, equation (25) expresses the probability amplitude of finding a proton or neutron of momentum K in the internal motion of the deuteron. Using the Hulthén wave function (22) for the deuteron, which is given by

$$\Psi_d(\vec{s}) = \frac{N}{s}\left[e^{-\alpha s} + e^{-\beta s}\right] \qquad (30)$$

and where

$$\alpha^2 = \frac{2m_d E_d}{\hbar^2} \quad , \quad \beta = 7\alpha \quad , \qquad (31)$$

equation (25) can be evaluated to give

$$D(K) = 4\pi N\left[\frac{1}{\alpha^2+K^2} - \frac{1}{\beta^2+K^2}\right] , \qquad (32)$$

where N is the normalizing constant.

D. The Bessel Function Factor, G(k).--The integral in equation (26) expresses the probability amplitude for capturing a neutron into the state $Y_l^m(\vec{r})$. By using the expansion

$$e^{ik\cdot r} = \sum (2l+1)\, i^l j_l(kr) P_l(\cos\theta), \tag{33}$$

where P_l is the Legendre polynomial and j_l is the spherical bessel function, the integral can be evaluated to give, taking the axis of quantization along the neutron momentum vector,

$$G(k) = \delta_{m0}\left[4\pi(2l+1)\right]^{1/2} i^l \int R_l(r)\, V(r)\, j_l(kr)\, r^2\, dr. \tag{34}$$

This equation can be put into an alternative form. This can be done by getting rid of the V(r) term. To do this we write the radial equations for the wave functions. For the radial wave function outside the well

$$\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right) + k^2 - \frac{l(l+1)}{r^2}\right] j_l(kr) = 0 . \tag{35}$$

Inside the well

$$\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right) - t^2 - \frac{l(l+1)}{r^2} - U(r)\right] R_l(r) = 0 . \tag{36}$$

Multiplying equation (35) by $R_l(r)$ and equation (36) by $j_l(kr)$ and subtracting, we obtain

$$\frac{R_\ell(r)}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}j_\ell(kr)\right) - \frac{j_\ell(kr)}{r^2}\frac{d}{dr}\left(r^2\frac{dR_\ell(r)}{dr}\right) + k^2 j_\ell(kr)R_\ell(r)$$

$$+\, t^2 j_\ell(kr)R_\ell(r) + U(r)\,R_\ell(r)\,j_\ell(kr) \;. \qquad (37)$$

We integrate over r, noticing that the first two terms drop out leaving us with

$$\frac{k^2+t^2}{2m_n'}\int_0^\infty j_\ell(kr)R_\ell(r)r^2\,dr = -\int_0^\infty R_\ell(r)V(r)j_\ell(kr)r^2\,dr \,, \qquad (38)$$

where m_n' is the reduced mass of the neutron. If we place this result in equation (34) we get the alternative form for G(k),

$$G(k) = -\delta_{m0}\left[4\pi(2\ell+1)\right]^{1/2} i^\ell \frac{(k^2+t^2)}{2m_n'}\int_0^\infty R_\ell(r)j_\ell(kr)r^2\,dr \;. \qquad (39)$$

E. Bhatia's Theory.--To obtain Bhatia's result for the cross-section we evaluate the integral in the G factor by making the assumption that the interaction of the neutron and the nucleus occurs only at some particular R which is equal to the nuclear radius plus the range of the nuclear forces; that is, it is a delta function interaction. Thus, for Bhatia's theory

$$G(k) \propto j_\ell(kr)\int V(r)\,R_\ell(r)\,r^2\,dr \qquad , \qquad (40)$$

or the cross-section is given by

$$\sigma(\theta) \propto \frac{k_p}{k_d} D^2(K) \left| j_l(kR) \right|^2 \quad . \qquad (41)$$

F. Butler's Theory.-- To obtain the Butler form for the stripping cross-section we again start with the G factor as it is given in equation (34). Following the same procedure outlined in part C to obtain the alternate form for the expression of G we arrive at the equation

$$\frac{R_l(r)}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}j_l(kr)\right) - \frac{j_l(kr)}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}R_l(r)\right)$$

$$+ \left(k^2+t^2+U(r)\right) j_l(kr)\, R_l(r) = 0 \ . \qquad (42)$$

However, instead of integrating this equation from zero to infinity we follow Butler's assumption and neglect the overlap integral from $r \leq r_0$ where r_0 is the nuclear radius. Thus we arrive at

$$\frac{1}{2m_n'}\int_{r_0}^{\infty}\left[R_l(r)\frac{d}{dr}\left(r^2\frac{d}{dr}j_l(kr)\right) - j_l(kr)\frac{d}{dr}\left(r^2\frac{d}{dr}R_l(r)\right)\right]dr$$

$$= -\frac{(k^2+t^2)}{2m_n'}\int_{r_0}^{\infty} j_l(kr)\, R_l(r)\, r^2\, dr \quad . \qquad (43)$$

The left hand side of the equation may be evaluated so that

$$\frac{1}{2m_n'}\left| r^2 R_\ell(r)\frac{d}{dr} j_\ell(kr) - r^2 j_\ell(kr)\frac{d}{dr}R_\ell(r)\right|_{r_0}$$

$$= -\frac{(k^2+t^2)}{2m_n'}\int_0^\infty j_\ell(kr)\,R_\ell(r)\,r^2\,dr \quad . \tag{44}$$

The right hand side is seen to be just the alternative form for G(k) so that, disregarding constants

$$G(k) \propto \left| R_\ell(r)\frac{dj_\ell(kr)}{dr} - j_\ell(kr)\frac{d}{dr}R_\ell(r)\right|_{r_0} . \tag{45}$$

In this case

$$R_\ell(r) = h_\ell'(itr) \quad , \tag{46}$$

where h_1' is the spherical hankel function of the first kind. Then,

$$\sigma(\theta) \propto \frac{k_p}{k_d} D^2(K)\left| j_\ell(kr)\frac{dh_\ell'(itr_0)}{dr_0} - h_\ell'(itr_0)\frac{d}{dr_0}j_\ell(kr_0)\right|^2 \quad . \tag{47}$$

G. Daitch and French Theory.--If the contribution to the overlap integral is not omitted for $r \leq r_0$, the Daitch and French form for the cross-section can be obtained. We define the quantity, which is the equivalent of a square potential well,

$$\overline{V}(r_0) = -\frac{\int_0^{r_0} j_\ell(kr)\,R_\ell(r)\,V(r)\,r^2\,dr}{\int_0^{\infty} j_\ell(kr)\,R_\ell(r)\,r^2\,dr} \quad . \tag{48}$$

Then it can be shown that the G factor is given by

$$G(k) \propto \left[1 - \frac{k^2+t^2}{2m_n'\,\overline{V}}\right] \int_{r_0}^{\infty} R_\ell(r)\,j_\ell(kr)\,r^2\,dr \quad . \tag{49}$$

Since the Butler form for G ignores the contribution of the overlap integral for $r \leq r_0$, the integral in equation (49) is precisely the Butler term for G. Thus the matrix element in the Daitch and French form is equivalent to that of Butler except for the multiplying factor

$$\left[1 - \frac{k^2+t^2}{2m_n'\,\overline{V}}\right] \quad . \tag{50}$$

This extra factor does not disturb the essential features of Butler's theory (the forward maximum) but it does remove one of the zeroes of the distribution. This occurs when the neutron momentum k equals an average wave number which a neutron could have when bound with binding energy

$$E_n = \frac{t^2}{2m_n'} \tag{51}$$

and with orbital angular momentum l_n in a well of radius r_o. The Daitch and French cross-section is then given by

$$\sigma_{DF}(\theta) = \left[1 - \frac{k^2+t^2}{2m'_n V}\right]\sigma_B \quad , \tag{52}$$

where σ_B is the cross-section obtained in the Butler case.

H. Serber Stripping.--It can be shown that the expression given in equation (27) is not inconsistent with Serber stripping, i.e., high energy stripping. Consider, for simplicity, the case of the Bhatia theory in which case the cross-section is given by

$$\sigma(\theta) \propto D^2 \left| j_l(kR) \right|^2 \quad , \tag{53}$$

where D(K) is given by

$$D(K) = \left[\frac{1}{\alpha^2+K^2} - \frac{1}{\beta^2+K^2}\right] \quad . \tag{54}$$

Since β is large ($\beta = 7\alpha$), we drop the second term in the brackets for an approximation. Since high energy stripping is concerned with energies of the order of hundreds of Mev, the energy of the emergent proton will be large. At this energy it is difficult to resolve the spectrum into a single energy. Thus, since the nuclear levels are relatively close together, as compared to the proton energies, the G term in the cross-section will be the sum of a number of adjacent levels which smooths out roughly to a constant. Serber cross-section becomes, approximately,

$$\sigma \propto \frac{k_p}{k_d}\left[\frac{1}{\alpha^2+K^2}\right]^2 , \qquad (55)$$

which is the form of the observed distribution in high energy stripping.

III. ENERGY DEPENDENCE OF THE ANGULAR DISTRIBUTIONS AND VALIDITY CRITERION FOR THE STRIPPING THEORIES

A. Validity Criterion.--The stripping cross-section as given in section II is expressed in the form

$$\sigma(\theta) \propto \frac{k_p}{k_d} D^2(K)\left|G(k)\right|^2 , \qquad (56)$$

where D(K) is the deuteron factor and G(k) is the neutron factor. Only those factors which do not affect the energy or angular dependence of the cross-section are omitted from equation (54). We can show that G(k) is actually a function of K. We start from the equations for K^2 and k^2 as given by equations (29) and (22),

$$k^2 = k_d^2 + c^2 k_p^2 - 2c k_p k_d \cos\theta \qquad (57)$$

$$K^2 = k_p^2 + \frac{k_d^2}{4} - k_p k_d \cos\theta , \qquad (58)$$

where $c = m_i/m_f$. If we multiply equation (58) by the factor 2c and subtract it from equation (57) we obtain

$$k^2 - 2cK^2 = \left(1 - \frac{c}{2}\right) k_d^2 + (c^2 - 2c) k_p^2 . \qquad (59)$$

Writing equation (58) in terms of the energies we get

$$k^2 - 2cK^2 = 2\left[\left(1 - \frac{c}{2}\right) m_d' E_d + (c^2 - 2c) m_p' E_p\right] , \qquad (60)$$

where m'_p and m'_d are the reduced masses of the proton and deuteron. From the conservation of energy equation for the reaction we know that

$$E_p = E_d + Q \quad . \tag{61}$$

Substituting this in equation (60) we obtain

$$k^2 - 2cK^2 = 2\left[\left(1 - \frac{c}{2}\right) m'_d E_d + (c^2 - 2c) m'_p (E_d + Q)\right] \tag{62}$$

or

$$k^2 - 2cK^2 = 2\frac{m_p}{m_T}\left\{\left[2\left(1 - \frac{c}{2}\right) m_i + m_f (c^2 - 2c)\right] E_d + m_f (c^2 - 2c) Q\right\} \quad , \tag{63}$$

where

$$m_T = m_d + m_i = m_f + m_p$$

is the total mass of the system.

Since $c = m_i/m_f$ we can write

$$k^2 - 2cK^2 = 2\frac{m_p}{m_T}\left\{\left[2\left(1 - \frac{c}{2}\right) m_i + 2 m_i \left(\frac{c}{2} - 1\right)\right] E_d + m_f (c^2 - 2c) Q\right\} \quad . \tag{64}$$

Thus we see that the coefficient of E_d cancels out and we are left with the relationship

$$k^2 - 2cK^2 = 2\, m_p \frac{m_f}{m_T} (c^2 - 2c) Q$$

$$= 2\, m_p' (c^2 - 2c) Q \quad . \qquad (65)$$

The neutron wave number, k, is then directly expressible in terms of K. The stripping cross-section can be written as

$$\sigma(\theta) \propto \frac{k_p}{k_d} \left| F(K) \right|^2 \qquad (66)$$

and for any particular reaction is only a function of k_p, k_d and K. If, however, we define a cross-section such that

$$\sigma' = \frac{k_d}{k_p} \sigma(\theta) \; , \qquad (67)$$

we see that

$$\sigma' \propto \left| F(K) \right|^2 \; . \qquad (68)$$

We now have a validity criterion for checking experiment with theory. If, instead of plotting σ versus θ as is done in angular distributions or σ versus E_d as is done in excitation functions, we plot σ' versus K, then all curves, either angular distributions or excitation functions for a given reaction should coincide. Also, since σ' is a function of K only we can investigate the energy dependence of the angular distributions by investigating K, which is a function of both E_d and Θ. An expression for the energy dependence of the width of the forward maximum can also be obtained in the same manner.

B. Energy Dependence of the Angular Distributions.--Because it is easier to use energies than wave numbers in actual practice the remainder of this work will be done in the variable E_k rather than K, where E_k is defined by

$$E_K = \frac{K^2}{2m_d'} \tag{69}$$

and is an energy associated with K.

If we pick an E_k on the σ' versus E_k curve this will be a constant no matter how E_d or θ varies. If the deuteron energy, E_d, is varied, θ will have to change to compensate for it. We make use of this fact in investigating the shift of the maximum of the angulur distribution (σ versus θ) with respect to a change in E_d.

We return to the equation for K^2 which is given by equation (58). We can rewrite this in terms of energies as

$$2m_d' E_K = K^2 = 2m_p' E_p + 2m_d' \frac{E_d}{4} - \sqrt{(2m_d')(2m_p') E_p E_d}\,\cos\theta \,, \tag{70}$$

where the primes denote the reduced masses. Using the conservation of energy equation

$$E_p = E_d + Q \tag{71}$$

we get

$$m_d' E_K = m_p'(E_d + Q) + m_d' \frac{E_d}{4}$$

$$- \sqrt{m_p' m_d'} \sqrt{E_d(E_d + Q)} \cos\theta \quad . \tag{72}$$

Simplifying and setting $a = \frac{m_f}{2m_i}$ we obtain

$$E_K = (a + \tfrac{1}{4})E_d + aQ - \sqrt{a}\sqrt{E_d(E_d + Q)} \cos\theta \, . \tag{73}$$

Since E_k is a function of both E_d and θ , we can write

$$dE_K = \frac{\partial E_K}{\partial \theta} d\theta + \frac{\partial E_K}{\partial E_d} dE_d \quad . \tag{74}$$

If we choose E_k to be a particular point, we have

$$0 = \frac{\partial E_K}{\partial \theta} d\theta + \frac{\partial E_K}{\partial E_d} dE_d \tag{75}$$

or

$$\frac{d\theta}{dE_d} = - \frac{\dfrac{\partial E_K}{\partial E_d}}{\dfrac{\partial E_K}{\partial \theta}} \quad . \tag{76}$$

From equation (73)

$$\frac{\partial E_K}{\partial E_d} = \left(a+\frac{1}{4}\right) - \sqrt{a}\,\frac{(2E_d+Q)}{\sqrt{E_d(E_d+Q)}}\cos\theta \tag{77}$$

and

$$\frac{\partial E_K}{\partial \theta} = \sqrt{a}\,\sqrt{E_d(E_d+Q)}\,\sin\theta\ . \tag{78}$$

Then,

$$\frac{d\theta}{dE_d} = -\frac{\left(a+\frac{1}{4}\right)-\sqrt{a}\left[\frac{2E_d+Q}{\sqrt{E_d(E_d+Q)}}\right]\cos\theta}{\sqrt{a}\,\sqrt{E_d(E_d+Q)}\,\sin\theta}\ . \tag{79}$$

Since the sine is always positive in the first or second quadrant, the denominator of (79) will always be positive. It is necessary to investigate only the numerator of equation (79) to determine the sign of the variation of θ with E_d.

Since the cosine is negative in the second quadrant, the numerator will always be positive for angles greater than 90°. Thus $\frac{d\theta}{dE_d}$ will be negative and the portion of the angular distribution beyond 90° will move in toward smaller angles for any increase of E_d.

For angles less than 90° it is simpler to plot the numerator for various angles and determine the energy at which it changes sign. To do this we set

$$A = \left(a + \tfrac{1}{4}\right) - \sqrt{a}\,\frac{2E_d + Q}{\sqrt{E_d(E_d + Q)}}\cos\theta \;. \qquad (80)$$

For convenience we change variables. If we set $x = \frac{E_d}{Q}$ we obtain

$$A = \left(a + \tfrac{1}{4}\right) - \sqrt{a}\,\frac{2x + 1}{\sqrt{x^2 + x}}\cos\theta \;. \qquad (81)$$

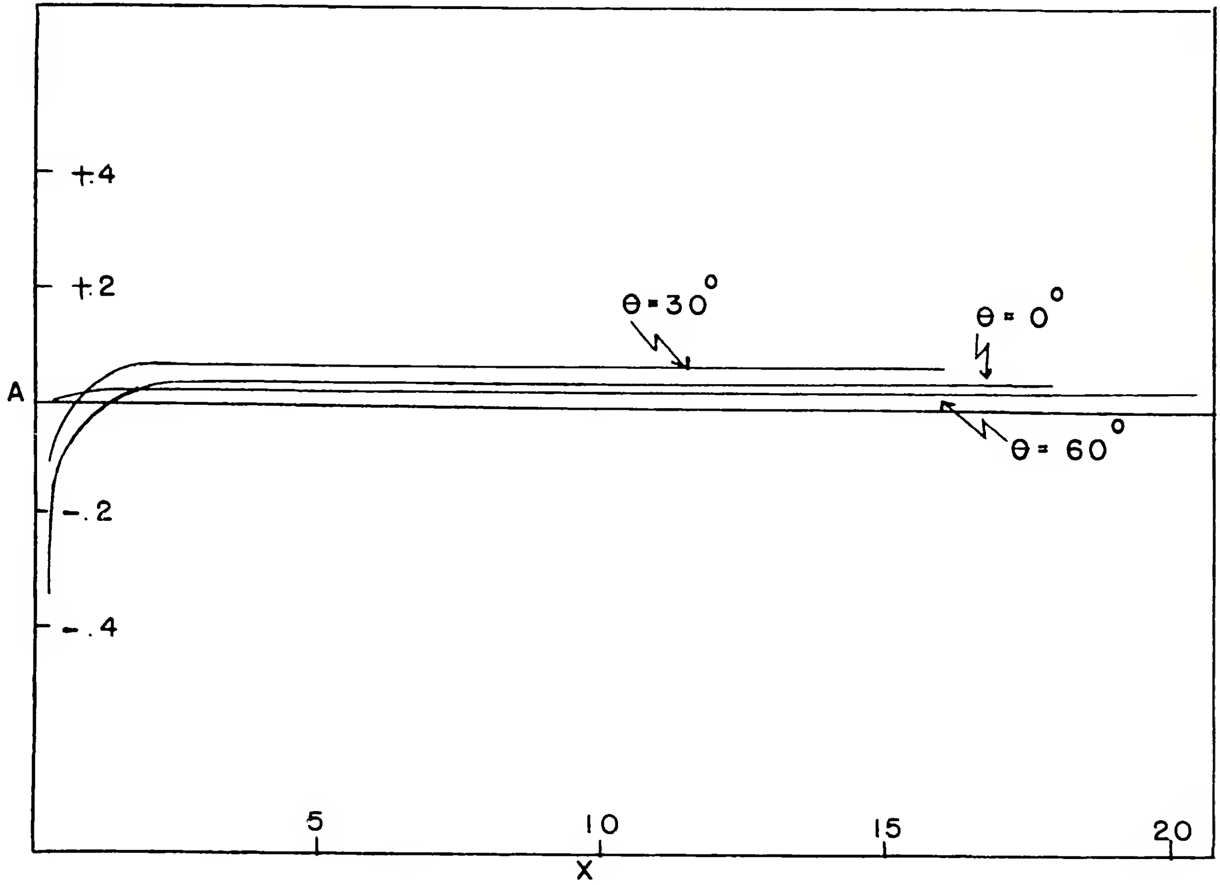
The variation of A with x for various angles is plotted in figure 3. As an approximation a is taken equal to one-half.

From the graph we see that for $\theta = 0^\circ$, A is negative until x equals one; that is, until $E_d = Q$. Thus $\frac{d\theta}{dE_d}$ would be positive (the angular distribution would tend to shift to higher θ) until E_d equalled Q, at which point A becomes positive and consequently $\frac{d\theta}{dE_d}$ negative. The curve would then tend to shift toward smaller θ with increasing E_d.

The same type of behavior occurs for $\theta = 30^\circ$ and $\theta = 60^\circ$ except that the cross-over point from minus to plus for A is pulled in toward lower deuteron energies (smaller x). The cross-over point, and hence the sign reversal for the $\theta = 30^\circ$ curve occurs at approximately x equals .4; the cross-over point for $\theta = 60^\circ$ occurs at approximately x equals .05.

Since for the C^{12} (d,p) reaction the forward peak (for the ground state) is between 25 and 30 degrees this means that the peak should move out in the angular distribution until $E_d = .4Q$ or 1.1 Mev.

Fig. 3.--Variation of A with respect to x for various angles.


+.4
+.2
A
-.2
-.4
θ=30°
θ= 0°
θ= 60°
5
10
15
20
X

For E_d greater than this value the peak should move in toward smaller angles.

Since angular distributions below about two Mev are unreliable due to high background from coulomb interactions, for all practical purposes the angular distribution curves would not be reliable in showing this reversing trend. There should be, a slight noticeable shift toward smaller angles for energies greater than about 2 Mev, but even this might be difficult to detect since the A versus x curve flattens quickly after crossing the axis.

C. Width of the Forward Maximum Versus E_d--To obtain an expression for the variation of the width of the forward maximum in the angular distribution with respect to E_d we consider the curve in Fig. 4. θ_1 and θ_2 are the angles associated with the half-value points. Returning to equation (73) we can solve for cos θ . We obtain

$$\cos\theta = \frac{(a+\frac{1}{4})E_d + aQ - E_K}{\sqrt{a}\,\sqrt{E_d(E_d+Q)}} . \tag{82}$$

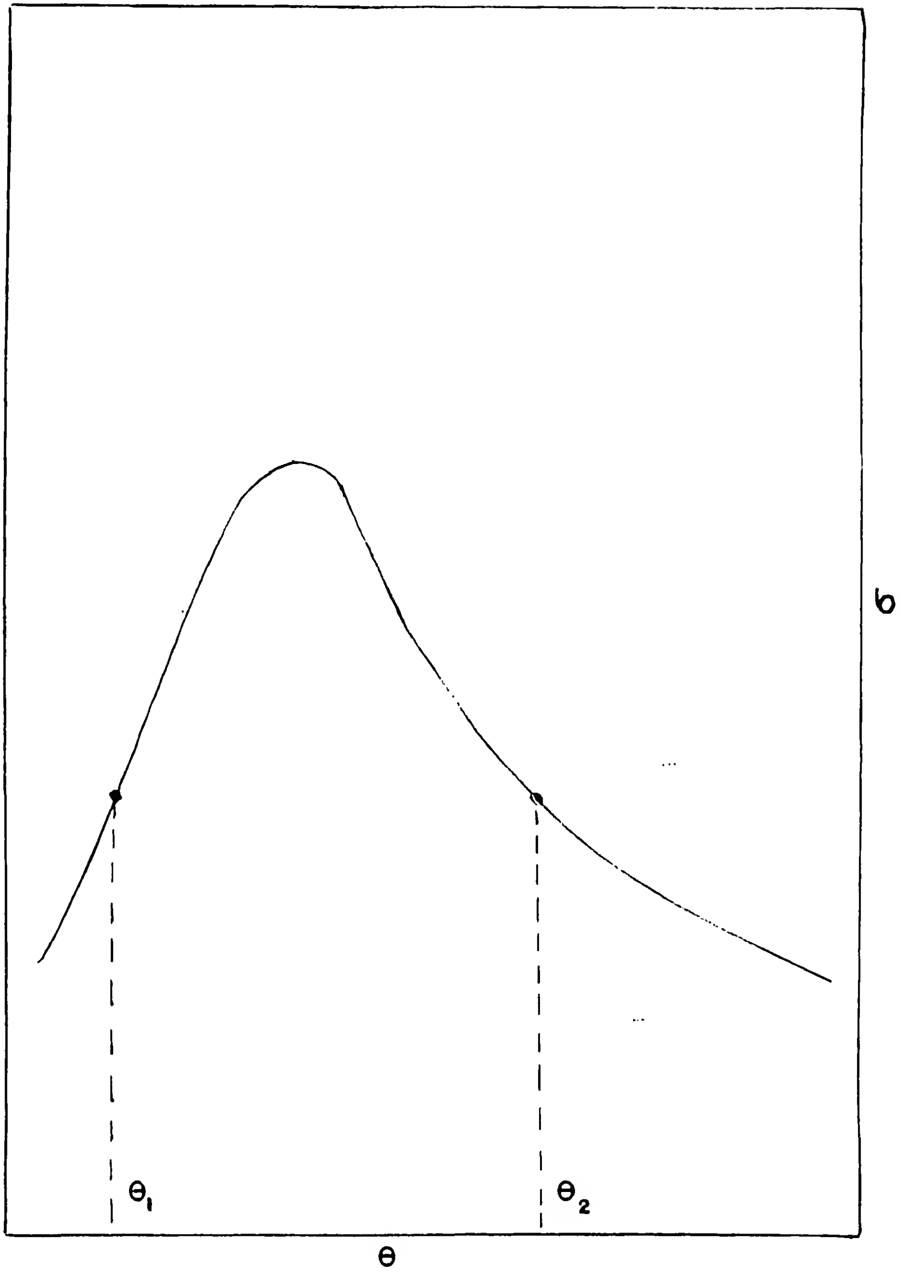
For θ_1 and θ_2 we can write

$$\cos\theta_1 = \frac{(a+\frac{1}{4})E_d + aQ - E_{K_1}}{\sqrt{a}\,\sqrt{E_d(E_d+Q)}} \tag{83}$$

and

$$\cos\theta_2 = \frac{(a+\frac{1}{4})E_d + aQ - E_{K_2}}{\sqrt{a}\,\sqrt{E_d(E_d+Q)}} . \tag{84}$$

Fig. 4.--Typical stripping peak in σ versus θ plot.



σ
θ₁
θ₂
θ

Subtracting,

$$\cos\theta_1 - \cos\theta_2 = -\frac{(E_{K_2} - E_{K_1})}{\sqrt{a}\sqrt{E_d(E_d+Q)}} \quad . \qquad (85)$$

The above expression is not an expression for the half-width but it does show how the width of the peak varies with E_d. Note that E_{k_1} and E_{k_2} are constants.

We see from equation (85) that the width of the peak is inversely proportional to the square root of $E_d(E_d+Q)$ and is always negative. Thus, the peak sharpens with increasing deuteron energies. To plot equation (83) we change to a new variable, $y = \frac{E_d}{Q}$, and let

$$W = \cos\theta_2 - \cos\theta_1 \quad .$$

Fig. 5 is a plot of the width, W, versus y where W has been normalized to the value it would have when $E_d = Q$; that is, y = 1 when $E_d = Q$. The forward peak sharpens quickly with increasing y (E_d). At y equal to approximately 2.5, the width of the peak is already half the value it would have at the Q value. By the time y reaches 8 the peak width is almost at a minimum.

D. Summary.--If the angular distributions and excitation functions obtained from stripping experiments are plotted in the new variables, $6'$ and K, they should coincide, if the simple stripping theories are correct. There should be only one characteristic curve of $6'$ versus K for any given reaction no matter what the deuteron energy.

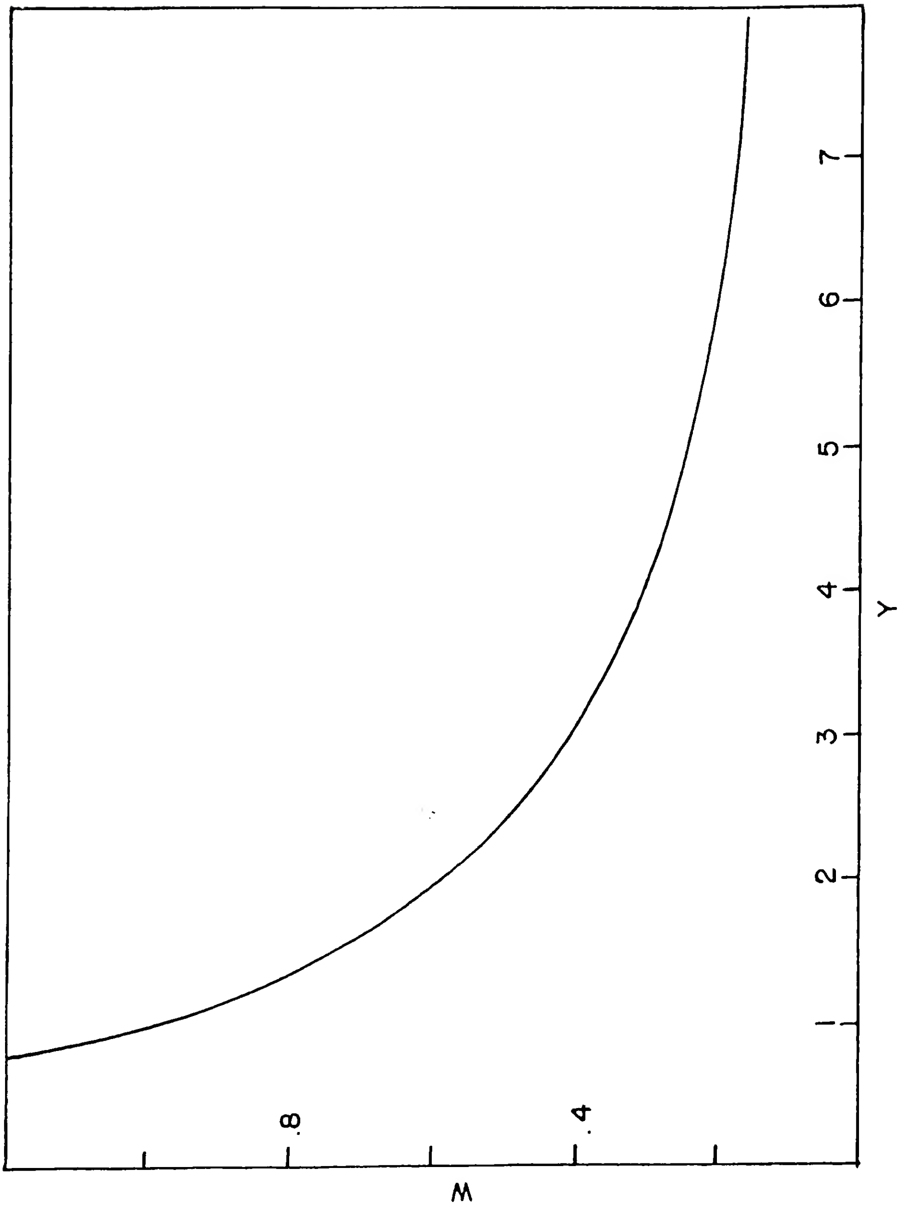

1
2
3
4
5
6
7
Y
.4
.8
W

Fig. 5.--Variation of the width of the stripping peak, W, with $y(E_d/Q)$.

The angular distributions plotted in the σ versus θ variables should be observed to shift with an increase in deuteron energy. For θ less than 90° the distribution should shift toward larger θ until the deuteron energy reaches the Q value. At that point the distribution should reverse and move in to smaller θ with increasing E_d. The portion of the angular distribution beyond 90° should always move in to smaller θ with an increase of E_d.

The width of the stripping peak should decrease with an increase in the deuteron energy. At deuteron energies around the Q value the width decreases rapidly with increasing E_d. At about $E_d = 8Q$ the width is approximately its minimum value.

IV. COMPARISON WITH EXPERIMENTAL RESULTS

A. Introduction.-- As was pointed out in section III, σ' is a function of K alone. Thus, for the same reaction, all experimental angular distributions plotted as σ versus θ for various values of E_d should coincide when replotted as σ' versus K. Also, the excitation functions when replotted in the σ' versus K variables should coincide with the angular distributions. Ideally then, there should be only one characteristic curve in the new variables for data taken in a reaction at any deuteron energy.

The reactions for which enough data was obtainable to provide a check with theory over an energy range were the C^{12} (d,p) ground state reaction, the O^{16} (d,p) ground state and the O^{16} (d,p) first excited state reaction.

The useable data for both the oxygen and carbon reactions was limited in quantity. Firstly, much of the work done was not expressed in absolute cross-sections which made it impossible to use for comparison purposes with the work of others. Also, data was needed at many deuteron energies so that a check with the validity criterion over a range of energies could be made, but very little work was done at energies higher than 8 Mev, except for the extreme range in the hundreds of Mev where Serber Stripping is characteristic. Much of the work done below 8 Mev was done near the coulomb barrier. At energies near the coulomb barrier it is difficult to interpret the data in terms of simple stripping. Compound nucleus effects and coulomb effects destroy the definite stripping characteristics of the angular distributions.

For example, the data of Berthelot, et al (23), for the oxygen (d,p) reaction is good but the deuteron energy range is from 1.66 to 2.2 Mev, which is too low; also, he does not give absolute cross-sections. Grosskreutz (24) data is given in absolute cross-sections but again the energy range is too low, between 1.05 and 2.71 Mev. Since the coulomb barrier for oxygen is about 3 Mev, this data is unreliable for stripping comparisons.

The oxygen data used was taken from the work of Stratton, et al, (25), Freemantle, et al (26), and Burge, et al (27). All curves except the 3.29 Mev curve of Freemantle and the 8 Mev curve of Burge are from Stratton. His work covers a range of deuteron energies between 2.25 and 3.8 Mev with an excitation curve being taken at 53° (center of mass coordinates). The data for the first excited state reaction of oxygen is also taken from his work.

Even less usable work was done for the carbon reaction than for the oxygen reaction. Again the difficulty is that the cross-sections were not given in absolute units or that the energy range of the deuteron was too low, as in the case of Koudijs, et al(28). His range of deuteron energies is from .2 to .65 Mev.

The carbon data used here is taken from the work of Bonner, et al (29), Holmgren, et al (30), and Rotblat, et al (31). The work of Bonner is excellent in that he works in the deuteron energy range from 1.8 to 6 Mev. Also, he takes excitation functions at three angles, 30, 90, and

150 degrees. However, only one of the excitation curves, that at 30 degrees, is used for comparison purposes.

B. Replotting of Data.--All data was transferred into the center of mass system. Except for the case of the 30° excitation curve of Bonner, all the cross-sections were already in the center of mass system, as were the scattering angles. The deuteron energies were always given in the lab system and had to be transferred into the center of mass system. The deuteron energies were translated to the new system by the use of the relation,

$$E_d = \frac{m_i}{m_d + m_i} E_{d_{LAB}} , \tag{86}$$

where E_d is now the center of mass energy.

In the case of the 30° excitation curve of Bonner, it was necessary to translate the lab cross-section into the center of mass system. To do this use was made of the relationship

$$\sigma_{LAB} = \frac{(1+\gamma^2+2\gamma\cos\theta)^{\frac{3}{2}}}{|1+\gamma\cos\theta|}\,\sigma . \tag{87}$$

The center of mass angle was obtained from

$$\gamma = \left(\frac{m_d m_p}{m_i m_f}\,\frac{E_d}{E_d+Q}\right)^{\frac{1}{2}} , \tag{88}$$

where

$$\gamma = \left(\frac{m_d m_p}{m_i m_f} \frac{E_d}{E_d + Q} \right)^{\frac{1}{2}} \qquad (89)$$

With all the data in the center of mass system, the experimental data was replotted. σ' was obtained from

$$\sigma' = \frac{k_d}{k_p} \sigma \quad , \qquad (90)$$

where

$$\left(\frac{k_d}{k_p} \right)^2 = \frac{2 m_i}{m_f} \frac{1}{1 + \frac{Q}{E_d}} \quad . \qquad (91)$$

Instead of K, E_k was used as the other variable. E_k is the energy associated with the momentum K and is given by

$$E_K = \frac{m_i}{2 m_f} (E_d + Q) + \frac{E_d}{4} - \sqrt{\frac{m_i}{m_f}} \sqrt{E_d (E_d + Q)} \cos \theta \quad . \qquad (92)$$

For an angular distribution of given E_d and Q, the values of were merely substituted in the equation and E_k obtained. Excitation functions were replotted using the same equations.

C. C^{12} Data.--The Q value of the ground state (d,p) reaction of C^{12} is 2.72 Mev. Using this value in equation (92), we obtain

$$E_K = .792\,E_d + 1.472 - .736\sqrt{E_d(E_d+Q)}\,\cos\theta . \quad (93)$$

From equations (90) and (91) we have

$$\sigma' = \left(\frac{1.85}{1+\frac{Q}{E_d}}\right)\sigma \quad . \qquad (94)$$

Fig. 6 is a replot of the excitation function for the carbon reaction taken from the work of Bonner and extends in energy range from $E_d = 1.8$ to $E_d = 6$ Mev at an angle of 30° in the laboratory system. Even though E_d was taken over a fairly large range of values, E_k, when computed, is over a short range. This is seen from equation (92). E_k is relatively insensitive to E_d at small angles. No stripping peak would be apparent due to the limited range of E_k. The total range of the excitation curve (Fig. 6) as compared to that of an angular distribution plot is shown in Fig. 7. The shaded section is the excitation function. The dashed line shown in Fig. 6 is the average over the resonances and, if the simple stripping theories were correct should follow the replotted angular distributions exactly.

It is immediately apparent that the stripping theories are inadequate since there are a number of resonances showing on the excitation curve. The simple stripping theories make no mention of resonances. The resonances shown in Fig. 6 are apparently from the compound nucleus N^{14}.

Fig. 6.--Excitation function of the C^{12} (d,p), ground state reaction, at $\theta = 30^{\circ}$ plotted as σ' versus E_K .

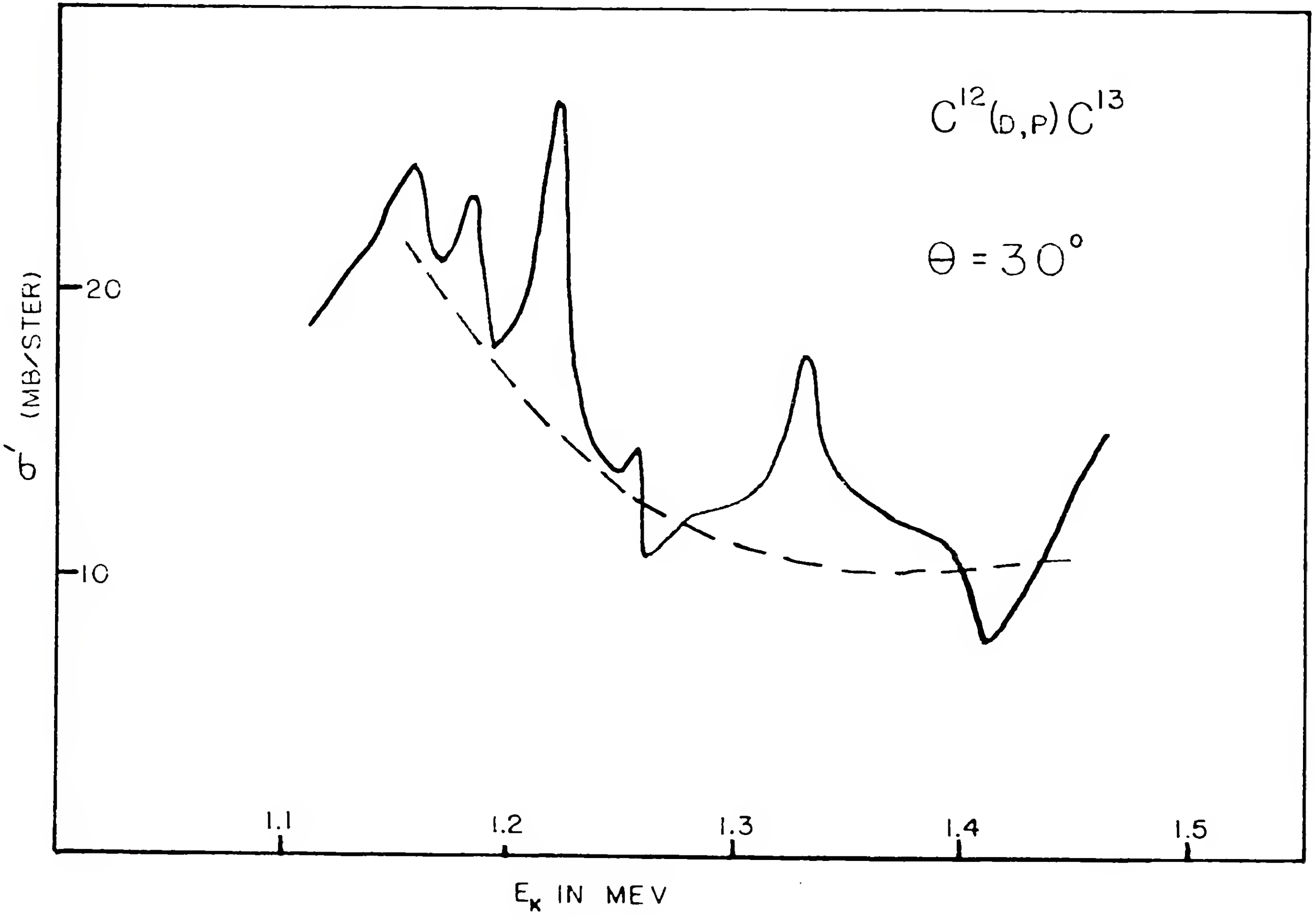

$C^{12}(D,P)C^{13}$
$\Theta = 30°$
σ′ (MB/STER)
20
10
1.1
1.2
1.3
1.4
1.5
E_x IN MEV

Fig. 7 is a replot of the 2.84 Mev, 3.02 Mev and 3.20 Mev curves of Bonner, et al. These curves bracket the 3 Mev resonance in N^{14} as is shown by the variation of their maxima with E_d. This 3 Mev resonance peak can be seen in Fig. 6. It is the large peak occurring at approximately E_k = 1.22 Mev. The dotted line in Fig. 6 corresponds to the bottom of the shaded portion seen in Fig. 7. Theoretically the excitation function should have the same slope as the angular distributions when plotted as σ' versus E_k. It does seem to exhibit this characteristic to some degree.

The three curves in Fig. 7 do not coincide as predicted by the simple stripping theories. At the same E_k, the forward maxima do seem to be closely aligned. The secondary maxima seem to shift out with higher E_d. This would tend to indicate a departure from stripping. These peaks, are probably due to other processes than stripping, most likely from compound nucleus formation. We are most interested in the forward or stripping peak.

Fig. 8 shows the 4.20 Mev, 4.09 Mev and 3.75 Mev curves of Bonner replotted in the new variables. These curves center about the 4.00 Mev resonance of N^{14}. The same general considerations that apply to Fig. 7 also apply here.

Bonner attempted to make an analysis of this 4.00 Mev resonance to determine the origin of the resonances. He assumed the cross-section to be the square of an amplitude that had two parts, a non-resonant part, σ_B , and a resonant, σ_r , which obeyed the Breit-Wigner formula. He assumed that there was a phase difference, $\delta(\theta)$, between the two

Fig. 7.--Angular distributions of the C^{12} (d,p), ground state reaction, at deuteron energies near 3 Mev plotted as σ' versus E_K.

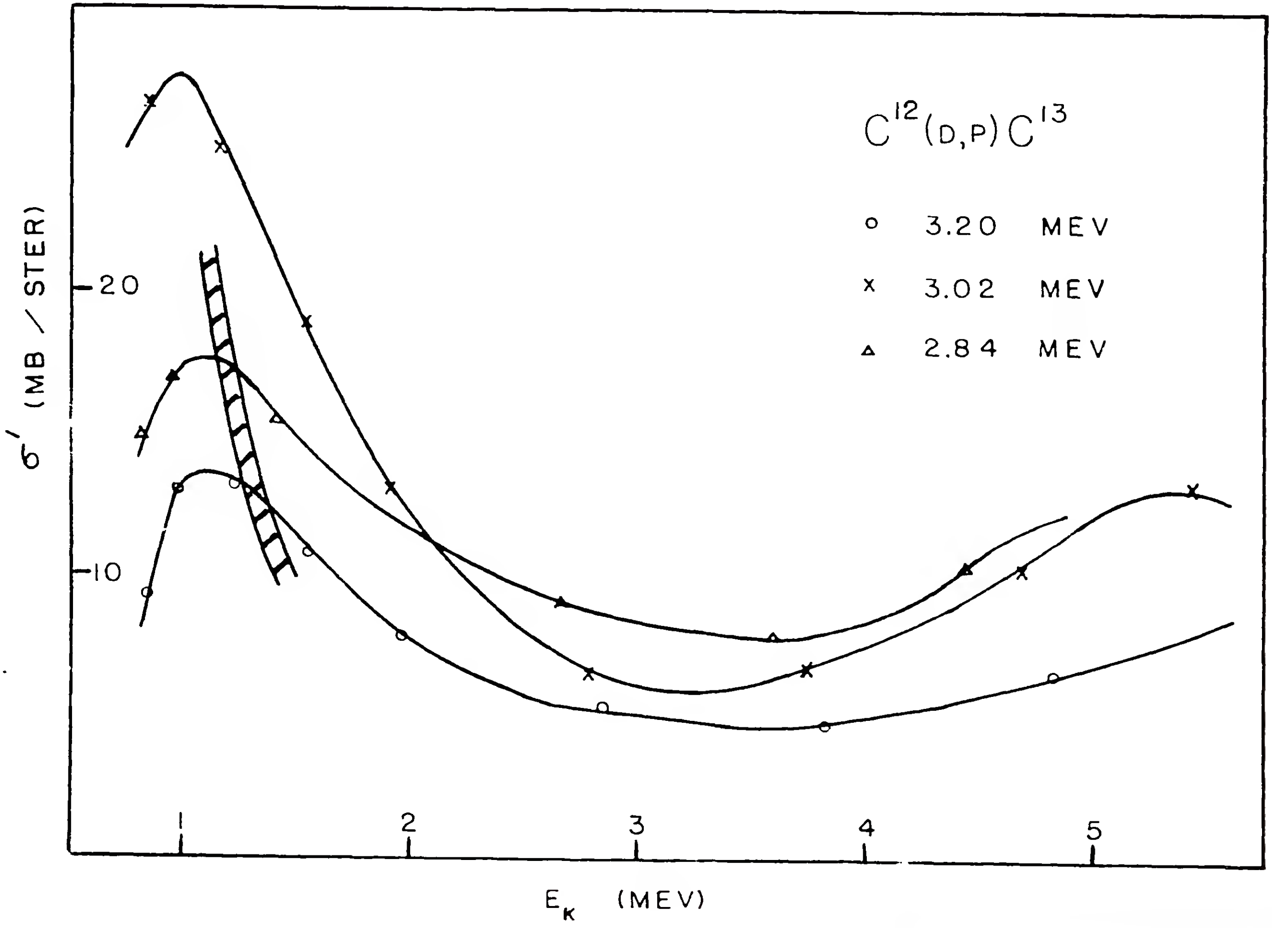
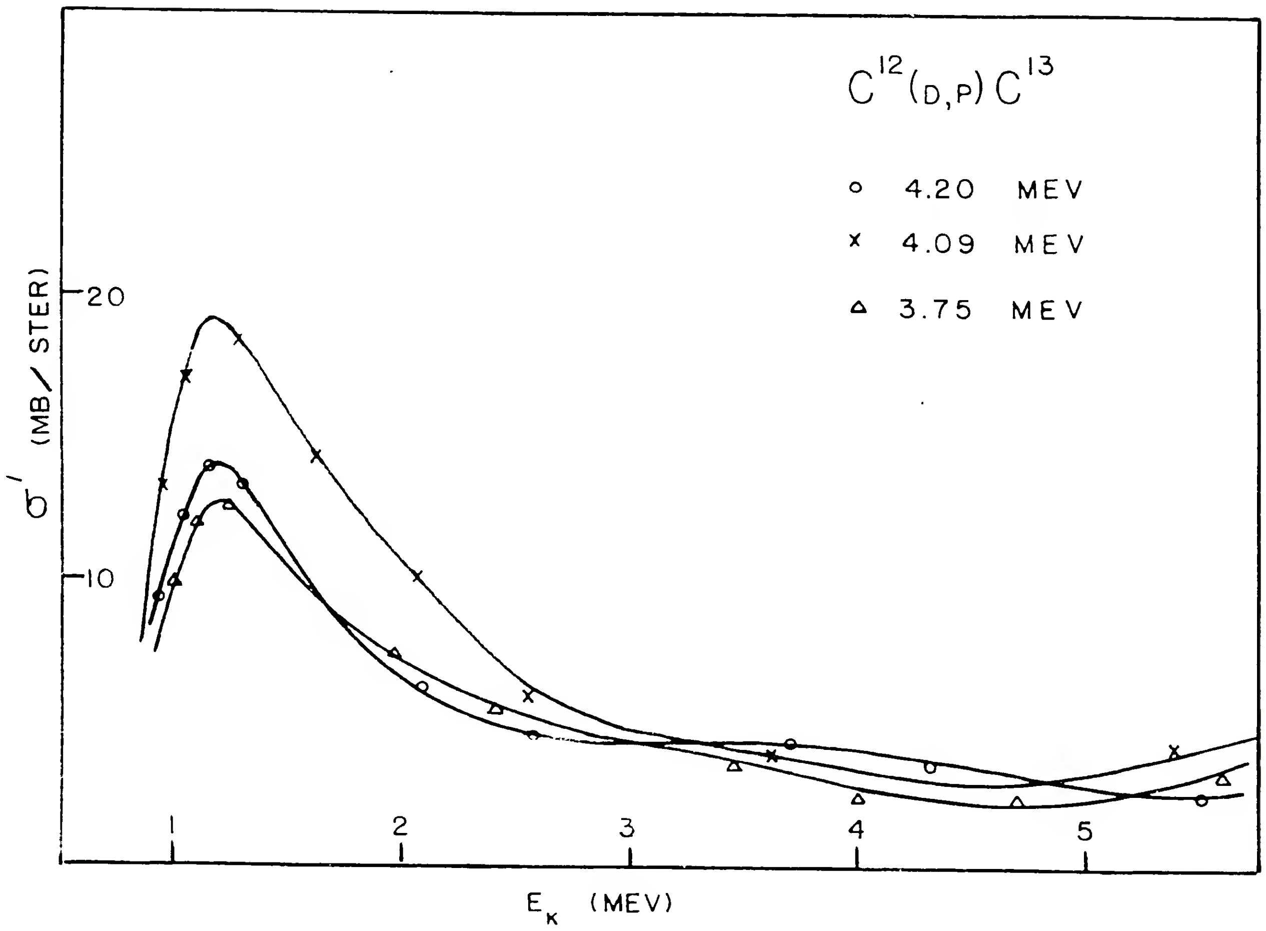

$C^{12}(D,P)C^{13}$
○ 3.20 MEV
× 3.02 MEV
△ 2.84 MEV
20
10
σ' (MB / STER)
1
2
3
4
5
E_K (MEV)

Fig. 8.--Angular distributions of the C^{12} (d,p), ground state reaction, at deuteron energies near 4 Mev plotted as σ' versus E_K.


$C^{12}(D,P)C^{13}$
○ 4.20 MEV
x 4.09 MEV
△ 3.75 MEV
σ' (MB/STER)
20
10
1
2
3
4
5
E_K (MEV)

components. According to Bonner the cross-section could be written as

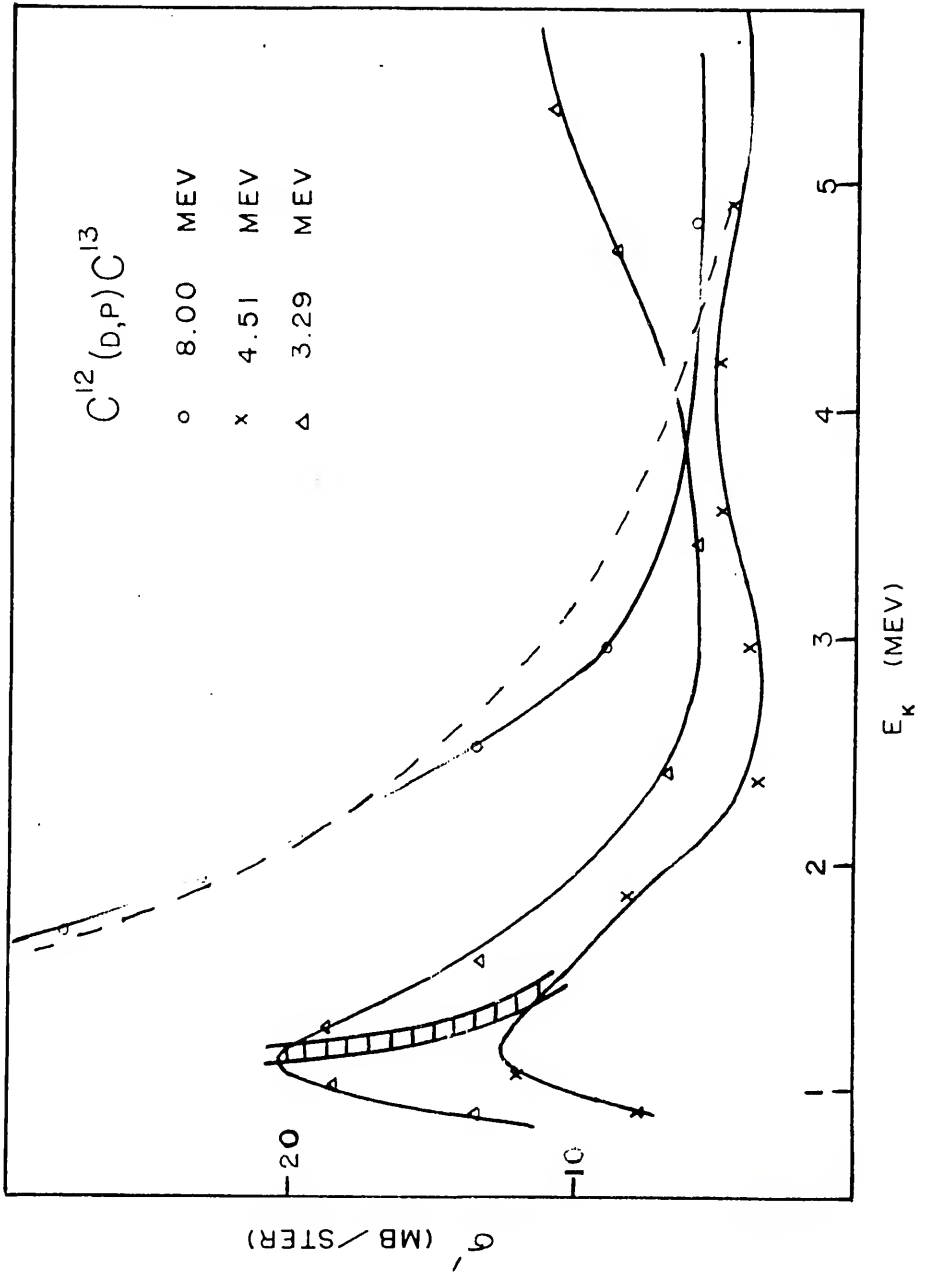
$$\sigma(E_d,\theta) = \left| \sigma_B^{\frac{1}{2}}(E_d,\theta)e^{i\delta(\theta)} + \tfrac{1}{2}i\Gamma \frac{\sigma_r(\theta)}{(E_d-E_r)+\frac{1}{2}i\Gamma} \right|^2 \qquad (93)$$

where E_R is the energy of the resonance, and Γ the width of the peak. An analysis of the 4.00 Mev resonance data of the carbon reaction indicated that the resonance was one in the compound nucleus N^{14}.

Fig. 9 shows a comparison of the 4.51 Mev curve of Bonner with the 8 Mev curve of Rotblat and the 3.29 Mev curve of Holmgren, et al. The 4.51 curve of Bonner and the 3.29 Mev curve of Holmgren were used for comparison purposes because both of them are off resonance. The intention was to obtain an energy spread to determine the extent of correlation of the curves over a range of E_d. Again the shaded section is the averaged-over excitation function, the one mentioned previously (see Fig. 6).

It is interesting to note that the 4.51 Mev curve of Bonner has a secondary maximum that seems to be out of place. The first suggestion of this maximum occurs in the 3.29 Mev curve in Fig. 7 and proceeds to become more pronounced with higher deuteron energy. Unfortunately there is no data available about the 4.51 Mev curve to determine the extent of the peak. At first glance it might seem that this maximum is due to some state mixing, but a Butler analysis was made to determine the existence of this maximum. According to the analysis the maximum is in approximately the proper place for a secondary stripping

Fig. 6.--Angular distributions of the C^{12} (d,p), ground state reaction, over a wide range of deuteron energies plotted as σ' versus E_K .


$C^{12}(D,P)C^{13}$
o 8.00 MEV
x 4.51 MEV
Δ 3.29 MEV
σ (MB / STER)
E_K (MEV)
20
10
1
2
3
4
5

maximum. It is probable that at lower deuteron energies other effects, predominately coulomb and compound nucleus effects, give a background that swamps this peak so that it is not noticeable.

The 8 Mev curve of Rotblat exhibits the characteristics of Serber stripping. A check was made to determine this. The Serber cross-section is given by, in the new variables,

$$\sigma'(K) \propto \left[\frac{1}{\alpha^2 + K^2}\right] , \tag{94}$$

where

$$\alpha = .23 \times 10^{13} \ cm^{-1} . \tag{95}$$

K^2 was expressed in terms of E_k by

$$K^2 = 2\, m_d' \, E_k . \tag{96}$$

The dotted curve shown in Fig. 9 is the theoretical curve for the 8 Mev data in terms of the Serber cross-section and normalized arbitrarily at the point $E_k = 2$ Mev. There is definite indication of Serber type stripping.

It is obvious from the data shown for the carbon (d,p) reaction that there is an energy dependent factor missing in the cross-section expression given by the simple stripping theories. Fig. 9 also indicates there should be a transition region between the Butler stripping and the Serber stripping. Most probably this transition comes about when the energy resolution of the experiment is poor such that the proton energies selected are not those due to a single level in the final nucleus.

D. O^{16} Data for Ground State Reaction.--The Q value for the ground state reaction is 1.92 Mev. Using this value, equation (92) for E_k becomes

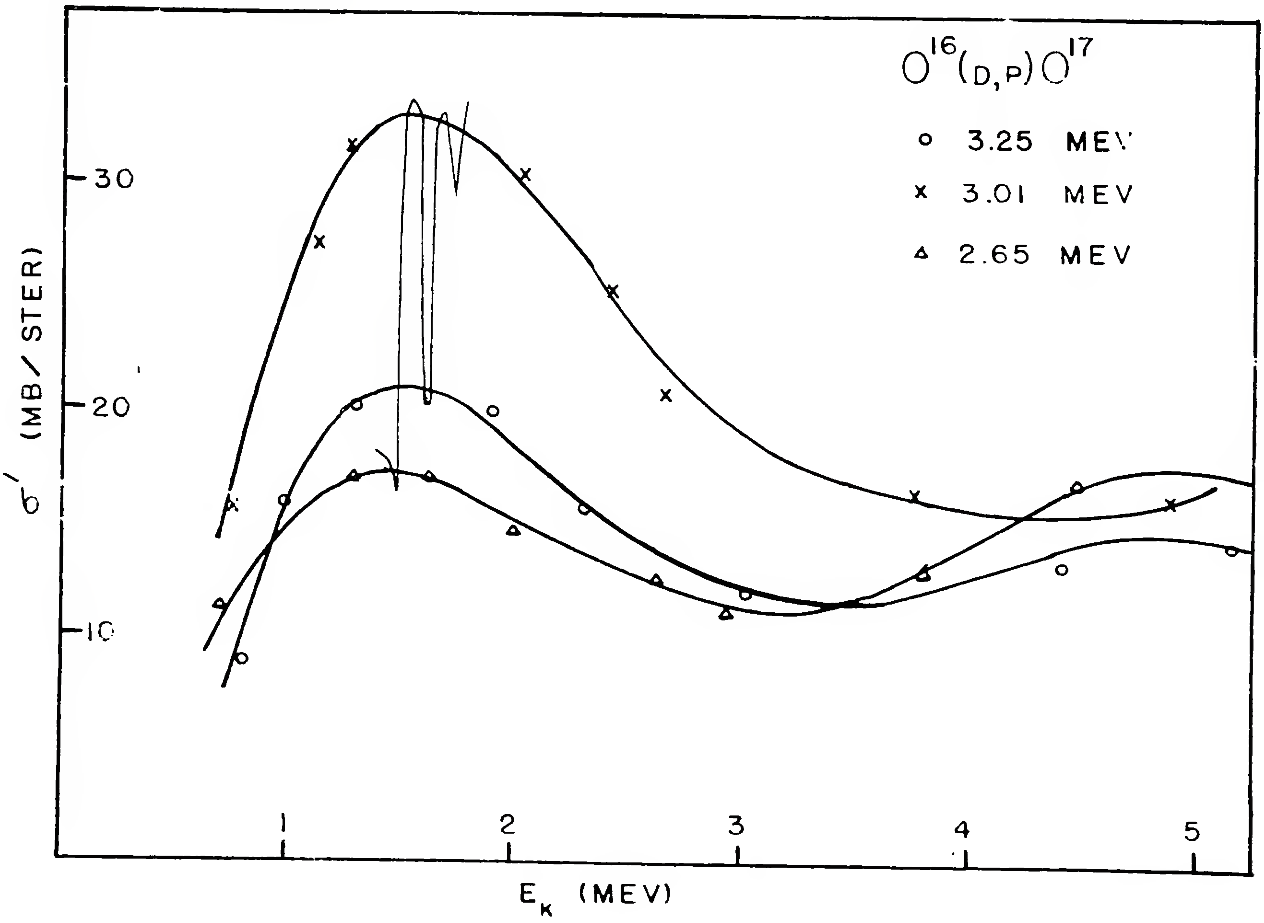
$$E_k = .7813E_d + 1.02 - .729\sqrt{E_d(E_d+Q)}\ \cos\theta . \tag{97}$$

The cross-section expression is

$$\sigma' = \left(\frac{1.88}{1+\frac{Q}{E_d}}\right)^{\frac{1}{2}}\sigma \ . \tag{98}$$

Fig. 10 shows the replotted angular distributions of the 2.65 Mev, 3.01 Mev and 3.25 Mev curves of Stratton, et al taken around the 3.00 Mev resonance of F^{18}. The small curve that cuts through the three angular distribution curves is the excitation function. Since the range of deuteron energies was small, 2.25 to 3.8 Mev, it was impossible to average over the resonances in the excitation curve for any great variation of E_k, but it is apparent that the peak of the angular distributions do rise and fall with the first maximum of the excitation curve

Fig. 10.--Angular distributions of the O^{16} (d,p), ground state reaction, at deuteron energies near 3 Mev plotted as σ' versus E_K.

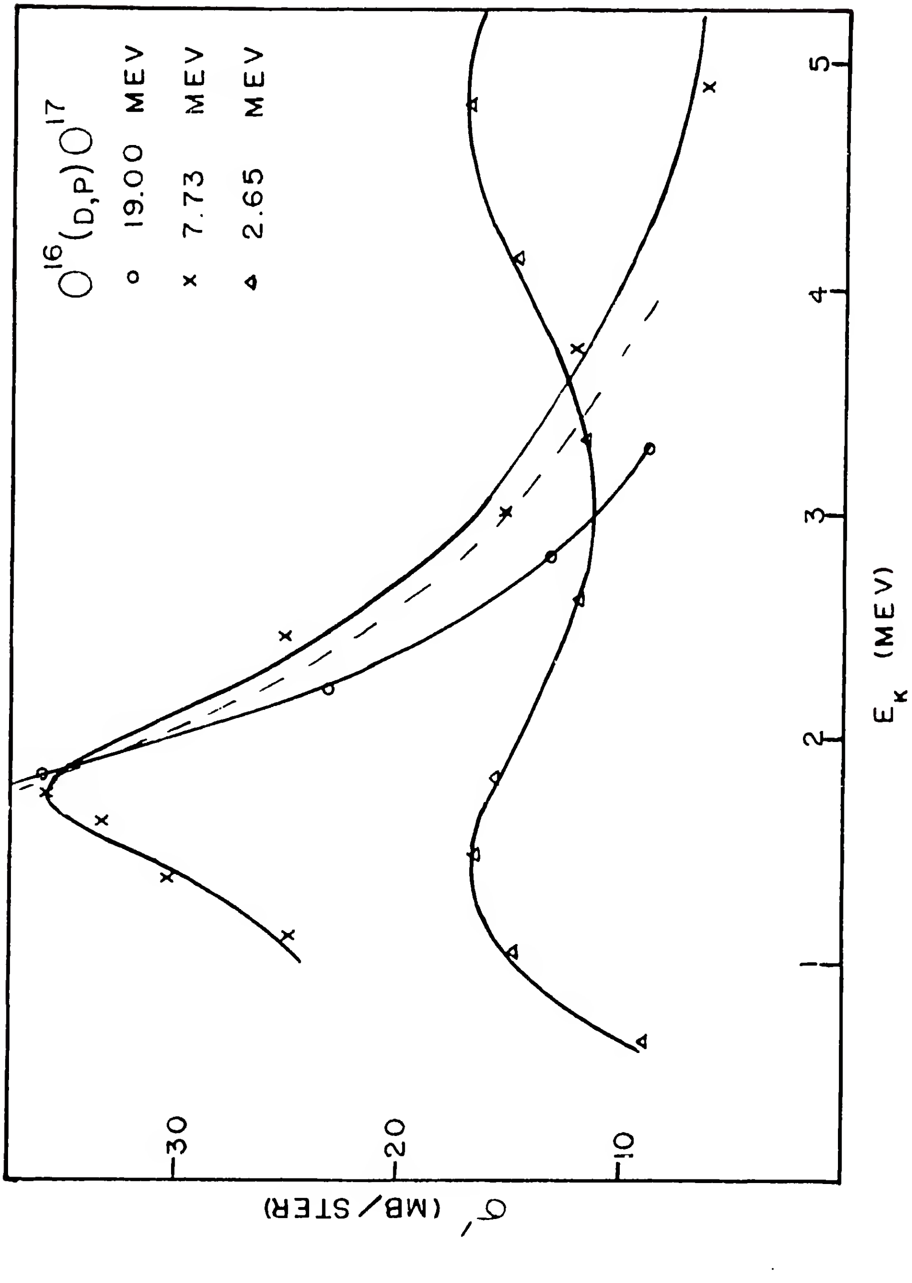

$O^{16}(D,P)O^{17}$
○ 3.25 MEV
× 3.01 MEV
△ 2.65 MEV
σ′ (MB / STER)
30
20
10
1
2
3
4
5
E_K (MEV)

which is the 3 Mev resonance of F^{18}. The primary maxima, the stripping maxima, are fairly well aligned indicating the predominance of stripping, but the secondary maxima are shifted relative to one another.

Fig. 11 shows a comparison of angular distributions over a range of values of E_d. The 19 Mev curve is taken from the work of Freemantle et al, the 7.73 Mev curve from the work of Burge et al, and the 2.65 Mev curve from the work of Stratton et al. The first maximum of the 7.73 Mev curve is shifted too far to the right which may be an error in the data of Burge rather than the peculiarity of the distribution. This same curve is reported in two different journals, once as an 8 Mev curve and once as a 7.73 Mev curve. The data for the 8 Mev curve was replotted and it does have its maximum at approximately the same F_k as the 2.65 Mev curve. This would seem more reasonable. It would be worth pointing out here that a Butler analysis could be made to fit either the 8 Mev curve or the 7.73 Mev curve, depending on the value of the nuclear radius taken. This would tend to indicate the arbitrariness of the simple stripping theories and points up the use of the new σ' and F_k variables. If there is consistency between the theory and experiment then there should be only one Butler analysis necessary for the angular distributions taken at any deuteron energies. Thus, poor data could be detected immediately. It would not coincide with data at other energies.

The 19 Mev curve exhibits some of the characteristics of Serber stripping. A check was made on this in the same manner as the one made for the 8 Mev curve of carbon. The results are shown by the dotted

Fig. 11.--Angular distributions of the O^{16} (d,p), ground state reaction, over a wide range of deuteron energies plotted as σ' versus E_K.


$O^{16}(D,P)O^{17}$
○ 19.00 MEV
× 7.73 MEV
△ 2.65 MEV
30
20
10
1
2
3
4
5
E_x (MEV)
σ (MB/STER)

line in Fig. 11. It is seen that there is much evidence of Serber stripping.

E. O^{16} First Excited State Data.--This reaction has a Q value of 1.04 Mev. Equation (92) becomes

$$E_K = .7813\,E_d + .553 - .729\sqrt{E_d(E_d+Q)}\,\cos\theta\ . \qquad (99)$$

The cross-section becomes

$$\sigma' = \left(\frac{1.88}{1+\frac{Q}{E_d}}\right)^{\frac{1}{2}}\sigma\ . \qquad (100)$$

Fig. 12 shows the replotted angular distributions taken from the work of Stratton et al, for the first excited state (d,p) reaction in oxygen. The three curves shown are the 3.01 Mev, the 3.25 Mev and the 3.43 Mev curves. These curves are centered around the 3.25 Mev resonance in Fl^{18}. There is much evidence of stripping here as is shown by the fact that all the curves coincide. It is also interesting to note that the maximum observed for the ground state reaction toward higher E_k (around 5 Mev) is missing. This would indicate predominate stripping for this reaction at this energy.

Fig. 13 shows three angular distributions for the first excited state reaction at various energies. The curves are the 19 Mev curve of Freemantle et al, the 7.73 Mev curve of Burge et al, and the

Fig. 12.--Angular distributions of the O^{16} (d,p), first excited state reaction, at deuteron energies near 3 Mev as plotted σ' versus E_K.

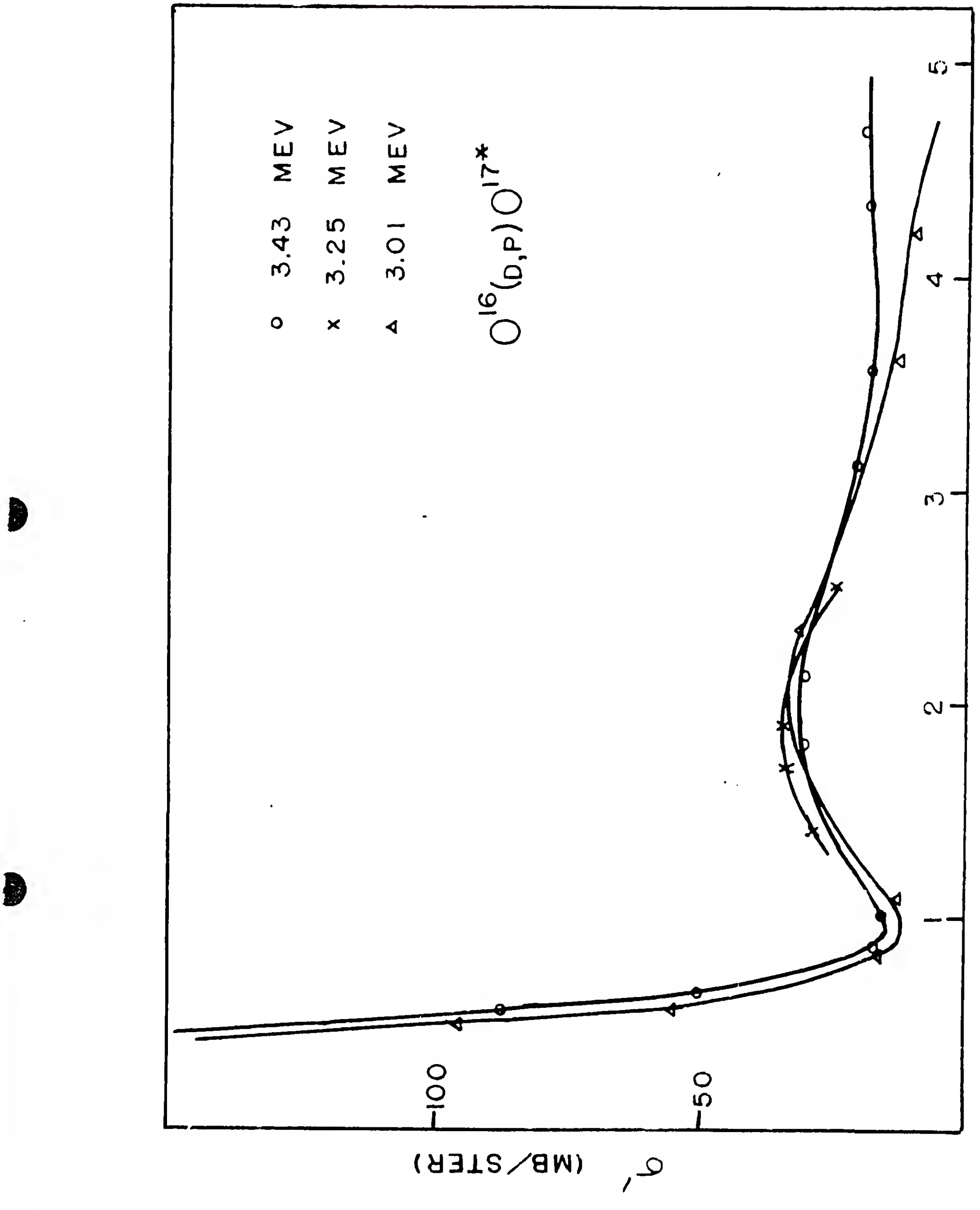
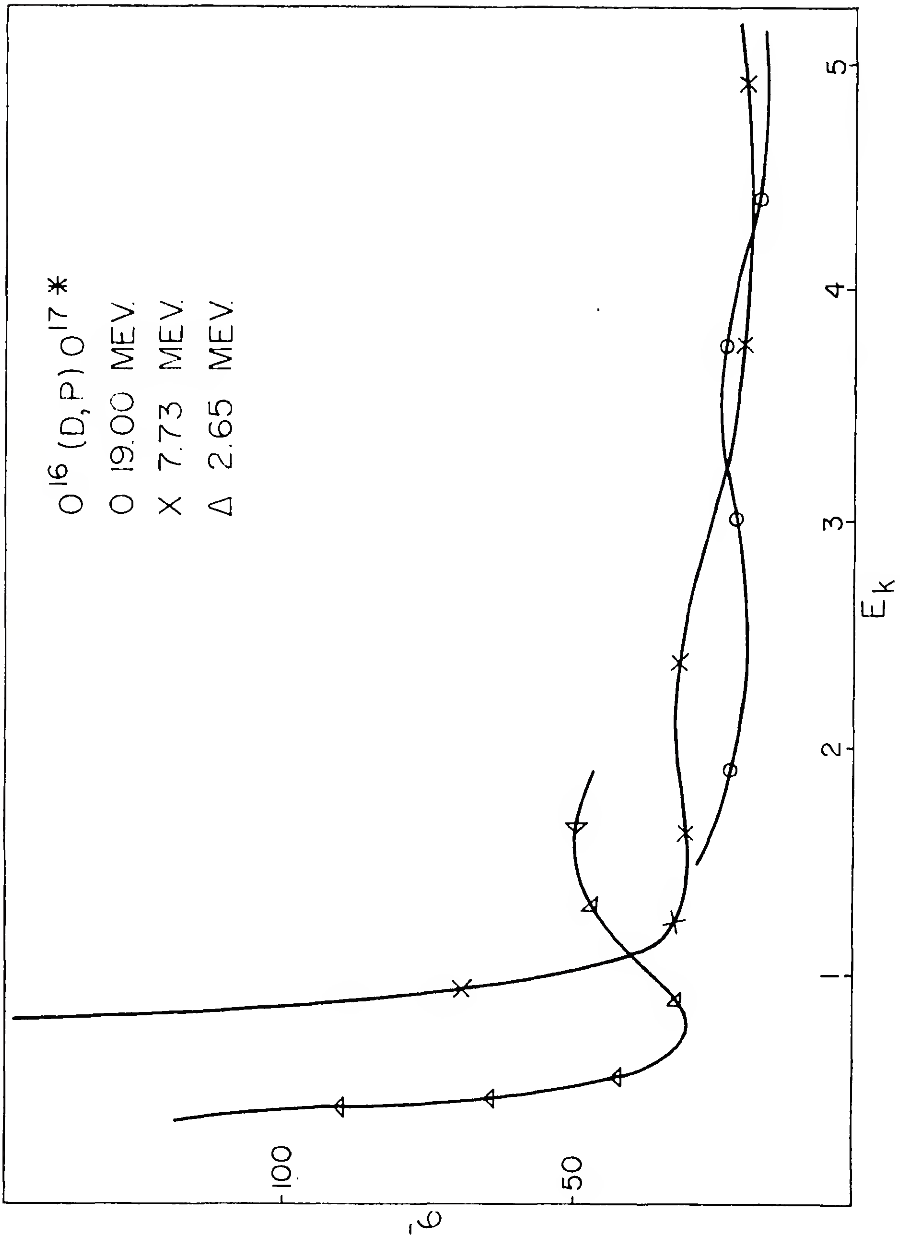

○ 3.43 MEV
× 3.25 MEV
△ 3.01 MEV
O16(D,P)O17*
EK (MEV)
1
2
3
4
5
50
100
σ (MB/STER)

Fig. 13.--Angular distributions of the O^{16} (d,p), first excited state reaction, over a wide range of deuteron energies plotted as σ' versus E_K .


$O^{16}(D,P)O^{17}$ *
O 19.00 MEV.
X 7.73 MEV.
Δ 2.65 MEV.
σ
100
50
E_k
1
2
3
4
5

2.65 Mev curve of Stratton et al. There is a noticeable shift of the curves toward higher E_k with an increase in E_d. As before we do not put too much faith in the 7.73 Mev curve of Burge. Unfortunately the 19 Mev data is not extensive enough to determine whether it would follow the 7.73 Mev curve or the 2.65 Mev curve. From the secondary maxima we would infer that there is a definite shift toward higher E_k with increasing deuteron energy. This would indicate there is another energy dependent factor missing from the cross-section. This is doubtful, however, since this does not show up for the other replotted data.

F. Analysis of the Experimental Curves.--An analysis was made of the 4.51 Mev angular distribution for C^{12} (d,p) reaction and the 3.43 Mev angular distribution curve for the ground state reaction in the O^{16} (d,p) reaction. Three theoretical analysis were made for each curve, one using the Butler theory, one using the Bhatia theory and one using the Daitch and French theory. As was pointed out in section II, the three theories are derived from the Born approximation for the cross-section and are similar except in their assumptions for the evaluation of the G(k) factor in the cross-section expression. Since the experimental curves were all replotted in terms of σ' and E_k, the theoretical curves were also plotted in these terms. Since each of the three theoretical versions of the theory predict that σ' is a function of E_k only, then one theoretical curve plotted in each reaction should, of course, fit all the data for that reaction.

Disregarding the constants, the cross-sections are given by, for the Butler theory,

$$\sigma' = \left[\frac{1}{\alpha^2+K^2} - \frac{1}{\beta^2+K^2}\right]^2 \left| j_l(kr)\,\frac{\partial h_l'(itr)}{\partial r} - h_l'(itr)\,\frac{\partial j_l(kr)}{\partial r}\right|^2_{r_0} , \tag{101}$$

for the Bhatia theory,

$$\sigma' = \left[\frac{1}{\alpha^2+K^2} - \frac{1}{\beta^2+K^2}\right]^2 \left| j_l(kR)\right|^2 , \tag{102}$$

and for the Daitch and French theory,

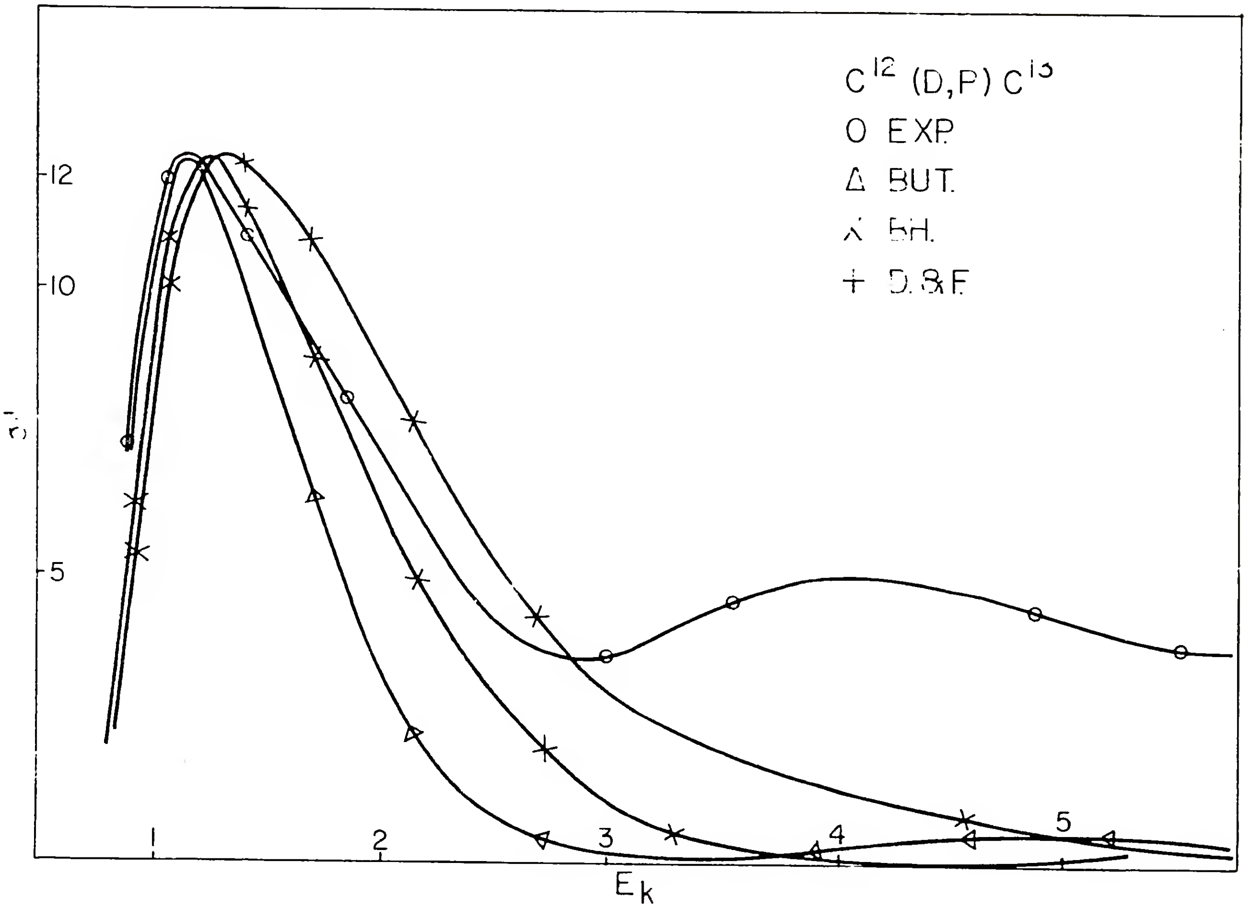
$$\sigma' = \left[\frac{1}{1-\left(\frac{k^2+t^2}{2m_n' V}\right)}\right]^2 \sigma'_{\text{BUTLER}} . \tag{103}$$

The significance and definition of the various parameters involved were given in section II.

G. Comparison of Carbon Data with Theory.--The 4.51 Mev angular distribution curve of Bonner was chosen as the experimental curve of the carbon reaction to analyze because of the secondary peak that seems out of place. The theoretical curves were normalized to the peak of the experimental curve so that the $\frac{k_d}{k_p}$ factor employed in the transformation from σ to σ' was not employed. A single radius, that fitting the Butler curve to the data, was used for the three theoretical curves. The Butler curve was chosen as the primary reference mainly because of the success of the theory in previous analysis.

The theoretical curves shown are the ones for which $l_n = 1$, which is the value that l should have according to the shell model theory since C^{13} is formed by adding the neutron in the 1p shell. Fig. 14 shows the comparison between the three theoretical curves and the experimental curve. Both the Bhatia curve and the Daitch and French curve are shifted toward higher E_k. The Butler theory seems to produce a valid curve only for a small region near the maximum of the forward peak. The Butler curve shows the secondary maximum of the experimental curve but it is shifted slightly toward higher E_k. The Daitch and French curve completely lacks the secondary maximum in the proper range. If the Bhatia curve were shifted to the left (larger nuclear radius), it would also exhibit the secondary maximum that the Butler curve shows, but the nuclear radius necessary to do this would be much larger than normally estimated. As it is, the radius necessary to fit the Butler curve to the experimental data is 6×10^{-13} cm, which is much too large compared to the value obtained from neutron scattering experiments. The Bhatia curve requires a radius of 7×10^{-13} cm to fit the experimental data. The radius necessary to fit the Daitch and French curve is approximately the same as that needed for the Bhatia curve. In the calculations of neutron cross-sections of

Fig. 14.--Comparison of the theoretical angular distribution curves and the 4.51 Mev experimental angular distribution curve of the C^{12} (d,p), ground state reaction, plotted as σ' versus E_K for a radius of 6×10^{-13} cm.


C^{12} (D,P) C^{13}
O EXP.
Δ BUT.
X BH.
+ D.&F.
12
10
5
1
2
3
4
5
E_k

14 Mev neutrons scattered from light elements, Coon et al (32), found that carbon should have a radius of 3.9 x 10^{-13} cm. Middleton, in analyzing the C^{12} (d,n) reaction found the radius of carbon to be 4.5 x 10^{-13} cm. This is the same order of magnitude as that found by Holt and Marsham from stripping reaction analyses. They indicate that the radius should be 4.1 x 10^{-13} cm.

The radius obtained from the 4.51 Mev curve analysis is higher than should be expected, it seems, even from a Butler analysis. This is not too difficult to understand, however, when it is noticed that the Butler radius is very sensitive to the position of the forward maximum. Thus, different data could give different results.

An interesting point presents itself in the Daitch and French analysis. The term which multiplies the Butler cross-section and which signifies the resultant difference between the theories can be used for an estimate of the nuclear well depth, assuming as Daitch and French does, a square well. This term is given by

$$\frac{1}{1-\dfrac{k^2+t^2}{2m_n'\overline{V}}} . \qquad (104)$$

The zero of this is

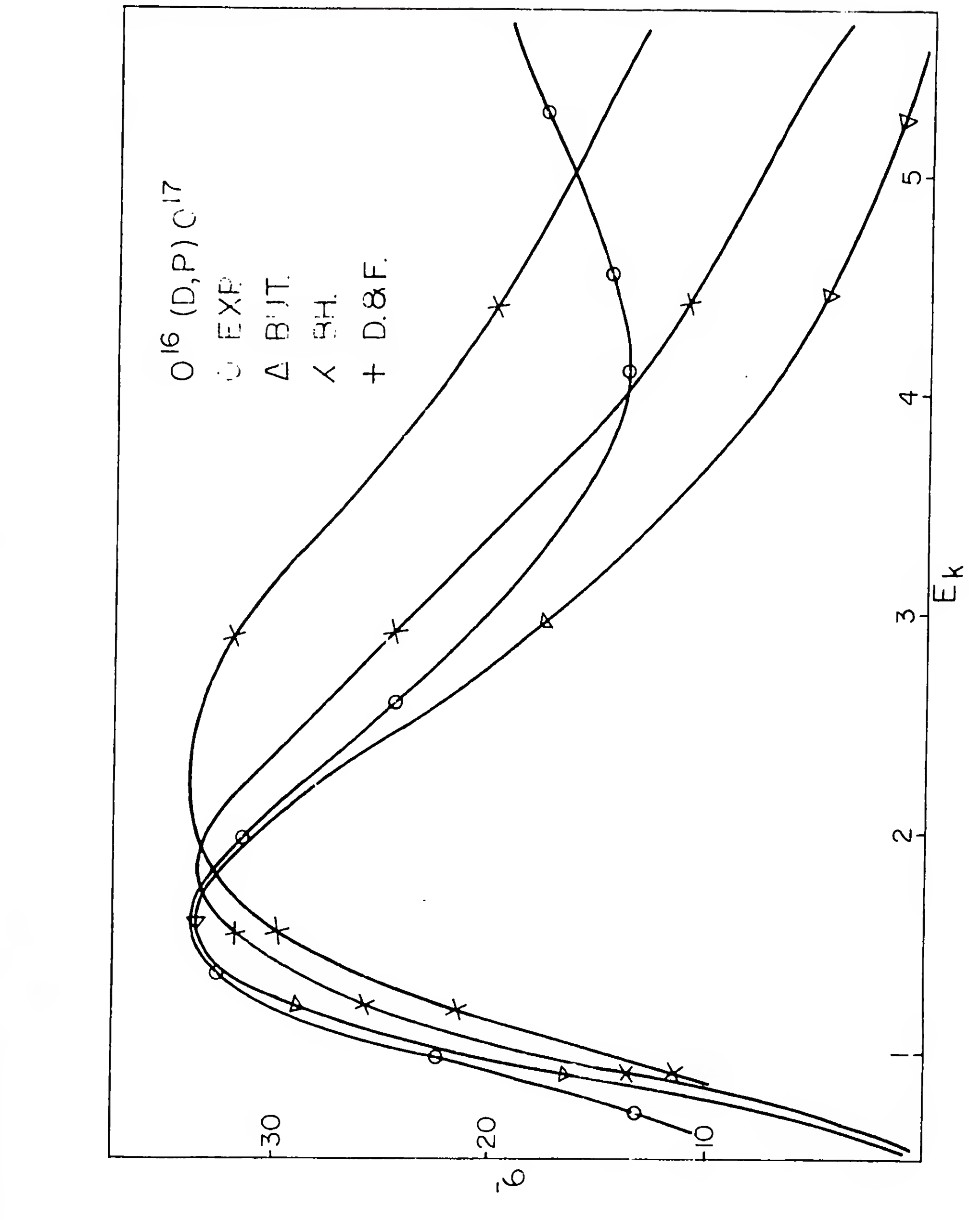
$$k_0^2 = 2m_n'\overline{V} - t^2 , \qquad (105)$$

where k_0 is the value for which the Butler theory has one of its zeroes (depending on the shell number). Knowing the value of k_o we can solve the equation for V. When, for the carbon (d,p) reaction the value of k_o is substituted, the potential well comes out to be about 10 Mev. This is certainly too shallow for a nuclear potential well and can only point up the fact that the theory is inadequate either in its assumptions or its evaluation.

H. Comparison of Oxygen Data with Theory.--The same type of theoretical analysis was done for the oxygen ground state and oxygen first excited state reactions as was done for the carbon. The experimental curve used for comparison in the ground state reaction was the 3.43 Mev curve of Stratton. The l_n value necessary to make the Butler curve fit the data is two, which is as it should be according to the shell model theory of the nucleus. The neutron added to O^{16} goes into the 1d state.

As can be seen from Fig. 15, with the Butler curve adjusted to experimental data the Bhatia and Daitch and French calculations are shifted toward E_k. The nuclear radius that was found necessary to fit the experimental data for the oxygen reaction is more reasonable than that necessary to fit the carbon data. The radius is 5×10^{-13} cm. To make the Bhatia curve fit it is necessary to have a radius of 6×10^{-13} cm. The Daitch and French curve requires a radius of 6×10^{-13}, also. The radius of oxygen, taken from the neutron scattering experiments of Coon is 4.3×10^{-13} cm. Holt and Marsham indicate that the radius should be 5.1×10^{-13} cm.

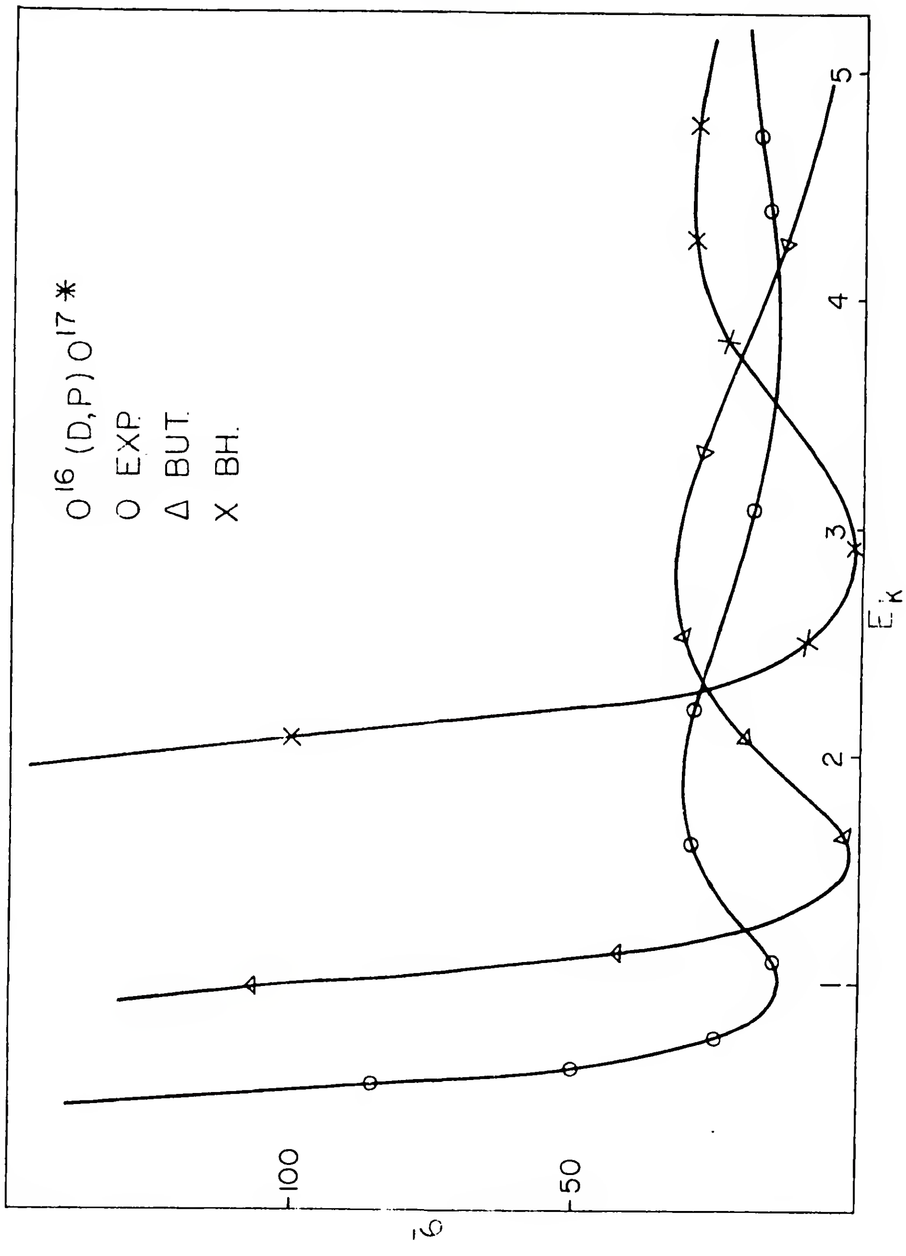
Fig. 15.--Comparison of the theoretical angular distribution curves and the 3.43 Mev experimental angular distribution curve of the O^{16} (d,p), ground state reaction, plotted as σ' versus E_K for a radius 5×10^{-13} cm.


O^{16} (D,P) O^{17}
○ EXP.
△ B.J.T.
× B.H.
+ D.&F.
σ
30
20
10
1
2
3
4
5
E_k

The secondary hump does not fall at all within the experimental curve, but this might be expected since at these low energies the secondary stripping maximum is swamped by the background. Also, coulomb effects destroy the predominant stripping characteristics of the curve. The secondary maximum exhibited here is probably due to the compound nucleus rather than stripping. It is seen in almost all stripping curves.

The analysis of the 3.43 Mev first excited state curve of Stratton presented somewhat of an anomaly on analysis. (See Fig. 16) The l value necessary to fit the experimental data is correct. It is $l_n = 0$, which corresponds to the neutron being added in an s state. It was assumed that the radius which fitted the ground state reaction should also fit the first excited state reaction, especially since the data was taken by the same author under the same conditions. This turned out to be untrue. The radius of 5×10^{-13} cm which is correct for the ground state reaction is too small for the first excited state reaction. Fig. 16 shows the comparison of the stripping theories, using the radius 5×10^{-13} cm. It was necessary to assume a radius of 6.5×10^{-13} cm to fit the experimental data. The Bhatia curve requires a radius of 9.5×10^{-13} cm to fit. The Daitch and French curve could not be drawn. The zero that is removed by the Daitch and French term in the expression for the cross-section could not be computed since it was not possible to obtain a value for k_o. In this case it is the second zero and not the first which is removed since the neutron is added in the 2s state; that is, the shell number is two.

Fig. 16.--Comparison of the theoretical angular distribution curves and the 3.43 Mev experimental angular distribution curve of the O^{16} (d,p), first excited state reaction plotted, as σ' versus E_K for a radius of 5×10^{-13} cm.


$O^{16}(D,P)O^{17*}$
O EXP.
Δ BUT.
X BH.
100
50
σ
1
2
3
4
5
E_K

V. CONCLUSIONS

The simple stripping theories agree with experimental results only in a limited region around the maximum in the forward direction of the angular distributions. There are more effects occurring than the simple stripping as is evidenced by the deviations of the experimental curves from the theoretical curves. These deviations are due to the exclusion of the compound nucleus and proton nucleus interaction in the simple theories.

As was shown in section II the best criterion for testing whether stripping predominates in a reaction is the plotting of both the angular distributions and excitation functions in the new variables σ' versus E_k. If simple stripping is predominant the curves should coincide. In the cases we have just investigated, the curves could not possibly coincide due to the non-inclusion of the resonances in the compound nucleus. Nevertheless, in the new variables the excitation function should fail within the extreme boundaries of the angular distributions.

We notice there is some alignment of the forward peaks in all the replotted angular distributions except for the first excited state of oxygen. There is a definite shift of the curves with energy. This would tend to indicate there is an energy dependent factor missing from the stripping theories, at least for this reaction.

The experimental data is not extensive enough to see much of a general trend in the comparison as shown in section III. This could be remedied in two ways. Firstly, the angular distributions should be taken at small deuteron energy intervals over a range of, say, from the Q value of the reaction investigated to the point where Serber stripping becomes noticeable. Also, the excitation function should be taken over this entire range of energies at several angles from zero to 180° so that a correlation could be made with the angular distributions when plotted in the σ' versus E_k variables. Secondly, if data can be taken only over a limited range of deuteron energies, it should be taken where stripping would be most noticeable. Since, as equation (85) shows, the width of the forward peak decreases with increasing deuteron energy the curves should be taken so that the width is at least twice as great as that at the Q value. Also, data should be taken so that the angular distribution curves are at energies well above the coulomb barrier and where the peak cross-section is large enough so that any secondary maximum due to stripping will not be swamped in the background.

In conclusion it should be pointed out that even though the simple stripping theories do allow one to assign spin-parities to nuclear levels and these agree well with the shell model of the nucleus, the theory should be used with some caution.

APPENDIX

A. Initial and Final Systems.--For a better understanding of the stripping process and the momentum vectors K and k we consider the initial and final systems of the stripping process.

In the initial system the deuteron is moving with momentum k_d and the target nucleus moving with momentum $-k_d$ with respect to the center of mass of the whole system. The velocity of the deuteron in this center of mass system is

$$\vec{v}_d = \frac{1}{m_d}\vec{k}_d = \frac{1}{2m_n}\vec{k}_d \tag{106}$$

and

$$\vec{k} = \vec{k}_d - \vec{k}_p \tag{107}$$

is the momentum.

Hence,

$$\vec{v} = \frac{1}{m_n}(\vec{k}_d - \vec{k}_p) \tag{108}$$

is the velocity of the neutron in the center of mass system. Now, the velocity of the neutron relative to the center of mass of the deuteron is

$$\vec{v}_n{}' = \vec{v}_n - \vec{v}_d \tag{109}$$

and

$$\vec{k}' = m_n \vec{v}_n - m_n \vec{v}_d = \vec{k}_d - \vec{k}_p - \tfrac{1}{2}\vec{k}_d = \vec{K} \,. \tag{110}$$

Thus, K is the momentum of the neutron as viewed from the center of mass of the deuteron. $|D(K)|^2$ is the probability that the deuteron moving with momentum k_d will emit a neutron with momentum

$$\vec{k} = \vec{k}_d - \vec{k}_p \,. \tag{111}$$

(This is virtual emission since the energy is not conserved though momentum is.

In the final system the proton is moving with momentum k_p and the product nucleus, B, is moving with momentum $-k_p$ relative to the center of mass of the whole system. The velocity of B (final nucleus) is

$$\vec{v}_B = -\frac{1}{(A+1)\,m_n}\,\vec{k}_p \,. \tag{112}$$

The velocity of the neutron is

$$\vec{v}_n = \frac{1}{m_n}\left(\vec{k}_d - \vec{k}_p\right) . \tag{113}$$

Then the velocity of the neutron relative to the center of mass of B is

$$\vec{v}_n'' = \vec{v}_n - \vec{v}_B \tag{114}$$

and

$$\begin{aligned}\vec{k}_n'' &= m_n(\vec{v}_n - \vec{v}_B) \\ &= \vec{k}_d - \vec{k}_p + \frac{1}{A+1}\vec{k}_p \\ &= \vec{k}_d - \frac{A}{A+1}\vec{k}_p = \vec{k} \qquad (115)\end{aligned}$$

is the momentum of the neutron as seen by the center of mass of B. Hence, $|G(k)|^2$ is the probability that A moving with momentum $-k_d$ will capture a neutron of momentum $k_d - k_p$ so that B moves off with momentum $-k_p$.

B. Deuteron Factor.--If the eigenfunction of a particle in the eigenstate is $\chi(\vec{s})$ then the probability that the particle in this state will be found with momentum K is

$$P(k) \propto \left| \int e^{-i\vec{K}\cdot\vec{s}}\, \chi(\vec{s})\, d\vec{s} \right|^2 \qquad (116)$$

since $e^{i\vec{K}\cdot\vec{s}}$ is the momentum eigenfunction. Thus the deuteron factor is proportional to the probability that the neutron in the deuteron will be found with K relative to the center of mass of the deuteron.

To make this clear, consider the deuteron as two particle systems. If the deuteron has momentum k_d its wave function is

$$e^{i\vec{k}_d\cdot\vec{r}_d}\, \chi(\vec{s}) \qquad (117)$$

where

$$\vec{r}_d = \tfrac{1}{2}(\vec{r} + \vec{r}_p)$$

$$\vec{s} = \vec{r} - \vec{r}_p \quad . \tag{118}$$

The momentum eigenfunction for the neutron and proton is

$$e^{i\vec{k}_p \cdot \vec{r}_p} \quad e^{i\vec{k}\cdot\vec{r}} \quad . \tag{119}$$

Thus the probability that the neutron and proton have k_p and k_n in the deuteron having momentum k_d is

$$P(\vec{k}_n, \vec{k}_p) \propto \left| \iint e^{-i\vec{k}_p \cdot \vec{r}_p} e^{-i\vec{k}\cdot\vec{r}} e^{i\vec{k}_d \cdot \vec{r}_d} \chi(\vec{s})\, d\vec{r}\, d\vec{r}_p \right|^2 . \tag{120}$$

Changing variables to

$$s = r - r_p$$

$$q = r_p \quad , \tag{121}$$

we obtain

$$P(\vec{k}, \vec{k}_p) \propto \left| \int e^{i(\vec{k}_d - \vec{k} - \vec{k}_p)\cdot\vec{q}}\, d\vec{q} \int e^{-i(\vec{k} - \frac{\vec{k}_d}{2})\cdot\vec{s}}\, d\vec{s} \right|^2 \tag{122}$$

The first integral is proportional to the Dirac delta function which is zero unless $k_d = k + k_p$, thus expressing the fact that momentum must be conserved. Therefore,

$$P(\vec{k}_n, \vec{k}_p) \longrightarrow P(\vec{k}_d - \vec{k}_p) \propto \left| \int e^{-i\vec{K}\cdot\vec{s}} \chi(\vec{s})\, d\vec{s} \right|^2$$

$$\propto P(K) \quad , \tag{123}$$

which is the same as before. We see that the motion of the deuteron makes no difference in computing P(K) as long as momentum is conserved.

C. The G(k) Factor.--Consider a free neutron of momentum k coming under the influence of a potential V(r) so as to be captured into the state with wave function $\psi(\vec{r})$. The wave function before capture is $e^{i\vec{k}\cdot\vec{r}}$. Hence the Born approximation for the capture probability is

$$P_{\substack{cap.\\ prob.}} \propto \left| \int \psi^*(\vec{r})\, V(r)\, e^{i\vec{k}\cdot\vec{r}}\, d\vec{r} \right|^2 \quad . \tag{124}$$

Here the center of mass of the system (neutron plus A equals to B) is taken at rest, so k must be the momentum of the neutron relative to the center of mass of B.

Consider the neutron and A as two particles whose center of mass is not at rest, but moving with momentum $-k_p$. Then, the initial wave function is

$$e^{i\vec{k}\cdot\vec{r}}\; e^{-i\vec{k}_d\cdot\vec{r}_A} \quad , \tag{125}$$

where the neutron has momentum

$$\vec{k} = \vec{k}_d - \vec{k}_p \tag{126}$$

and the target nucleus has momentum $-k_d$.
The final wave function is

$$e^{-i\vec{k}_p\cdot\vec{r}_B}\; \varphi(\vec{r}) \quad , \tag{127}$$

where

$$r_B = \frac{A r_A + r}{A + 1} \tag{128}$$

is the center of mass of the final nucleus, and r_A is the position vector of A. The matrix element is then

$$\int e^{i\vec{k}_B\cdot\vec{r}_B}\; \varphi^*(\vec{r})\; V(\vec{r})\; e^{i\vec{k}\cdot\vec{r}}$$

$$\times\; e^{-i\vec{k}_d\cdot\vec{r}_A}\; d\vec{r}\; d\vec{r}_A \quad . \tag{129}$$

Let

$$r = r_n - r_A \tag{130}$$

and

$$q = r_A \tag{131}$$

as before. The matrix element becomes

$$\int e^{i(\vec{k} + \vec{k}_p - \vec{k}_d)\cdot\vec{q}}\, d\vec{q}$$

$$X \int \overset{*}{\varphi}(\vec{r})\, V(r)\, e^{i\left(\vec{\boldsymbol{k}}_d + \frac{\vec{k}_p}{A+1}\right)\cdot\vec{r}}\, d\vec{r} \quad . \tag{132}$$

The first factor is proportional to the delta function $\delta(\vec{k}+\vec{k}_p-\vec{k}_d)$ requiring $\vec{k}=\vec{k}_p-\vec{k}_d$. Thus, we have,

$$\int \overset{*}{\varphi}(\vec{r})\, V(r)\, e^{i\vec{\boldsymbol{k}}\cdot\vec{r}}\, d\vec{r} \quad , \tag{133}$$

where

$$\vec{\boldsymbol{k}} = \vec{k} + \frac{\vec{k}_p}{A+1} = \vec{k}_d - \frac{A}{A+1}\vec{k}_p \quad . \tag{134}$$

Hence $|G(\boldsymbol{k})|^2$ is proportional to the probability that the target nucleus moving with momentum $-k_d$ will capture a neutron moving with momentum $\vec{k} = \vec{k}_d - \vec{k}_p$ to form the final nucleus moving with momentum $-k_p$. Momentum is conserved in each step of the reaction, but energy is conserved only between the initial and final states. That is, energy "borrowed" to form the neutron from the deuteron with momentum $k = k_p - k_d$ is paid back when the neutron is captured.

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BIOGRAPHICAL SKETCH

Harold Arthur Moore was born in Brackettville, Texas on February 4, 1925. He was awarded the B. S. degree in physics by St. Mary's University, San Antonio, in 1944. He served in the Army of the United States until July, 1946, at which time he entered Washington University in St. Louis, Missouri. He was awarded the M. S. degree in physics from Washington University in June, 1948, and joined the staff of the Missouri Research Labs in St. Louis. He remained with them in the capacity of Junior Physicist until 1951 at which time he entered the University of Florida to do graduate work in physics. In 1952 he joined the staff of the C-2 Department as an instructor and remained with the Department until September, 1954, at which time he returned to his graduate studies in theoretical physics leading to the degree of Doctor of Philosophy. He is a member of Sigma Xi, Pi Mu Epsilon and the American Physical Society.

This dissertation was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of that committee. It was submitted to the Dean of the College of Arts and Sciences and to the Graduate Council, and was approved as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August 11, 1956

C. F. Byers
Dean, College of Arts and Sciences

L. E. Grinter
Dean, Graduate School

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